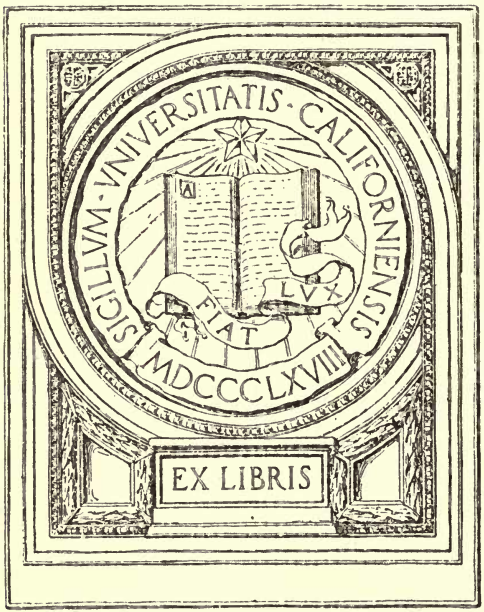
UC-NRLF
$B 276 929


SIGILLVM · VNIVERSITATIS · CALIFORNIENSIS
LVX
FIAT
MDCCCLXVIII
EX LIBRIS

MATHEMATICS

FOR

MACHINISTS

BY

R. W. BURNHAM, M.A.

Teacher, Erasmus Hall High School, Brooklyn, N. Y.;
Instructor, Evening Machine Classes, Pratt Institute;
Co-ordinator in Co-operative Work, High Schools of The City of New York

FIRST EDITION

TOTAL ISSUE, SEVEN THOUSAND

NEW YORK
JOHN WILEY & SONS, INC.
LONDON: CHAPMAN & HALL, LIMITED
1915

11/3/20

TJ1165
B9

Copyright, 1915
BY
R. W. BURNHAM

PRESS OF
BRAUNWORTH & CO.
BOOK MANUFACTURERS
BROOKLYN, N. Y.

PREFACE

In an experience of more than ten years in teaching machine-shop work to evening classes of men and boys actually engaged in the trade, the author has observed a decided lack of mathematical knowledge among ordinary mechanics. Many leave school from the grammar grades. Any mathematical training that they once may have received is, therefore, so far behind them by the time they are well started in their trade that it has practically been forgotten. About all that has been retained is a fair understanding of addition, subtraction, multiplication and division. For such, the natural starting point for a further knowledge of mathematics is the study of fractions.

Beginning with fractions, this book aims to give, in elementary form, an explanation of the calculations most frequently occurring in machine-shop work. The treatment has been made as simple as possible, in some places almost too simple, perhaps, with the desire to put the explanation in such a form as to be easily understood.

Many mechanics are mistakenly impressed with the extent of their mathematical knowledge. If they are able to take some formula from a handbook and apply it by rule of thumb to a particular calculation they are called upon to make, they are entirely satisfied. Experience has shown that such knowledge is often so shallow that a mere change in the letters of a formula will so disguise its meaning that the mechanic fails utterly even to recognize

423685

it; much less has he the ability to apply the changed formula to any calculation.

The aim of this book is to discourage, as far as possible, the practice of using a formula mechanically with no thought for its meaning. Wherever possible, the attempt has been made to show the steps in a calculation in logical order. If these steps are once understood, there is no objection to using a formula as a final rule reached; but the opposite practice of blindly following a rule, without knowing anything about its source or meaning, is not conducive to clear thinking; and it is the ability to think clearly which makes the master mechanic.

It is believed also that the material here presented and the method of treatment will be found well adapted to day trade, continuation and cooperative classes, and that teachers of such classes will find this text helpful in securing a closer coordination between the shop and class-room work.

The author is indebted to Mr. J. W. Burley, Head of the Machine Department, Pratt Institute, Brooklyn, N. Y., for many valuable suggestions in the text, and to Mr. L. P. Burnham, Statistician, Public Service Commission, New York City, who wrote the chapter on Business Organization and contributed many of the problems.

It is fully realized that there is room for decided improvement in all parts of the book, and criticism with regard to the methods used and the explanations given will be warmly welcomed. If the small beginning made here becomes an incentive to anyone to strive to rise to higher things, the author will feel well repaid for the time and effort he has spent in collecting the material.

R. W. Burnham.

Brooklyn, N. Y.
April, 1915.

CONTENTS

CHAPTER I

COMMON FRACTIONS

PAGE

Fractions.—Lowest Terms.—Mixed Numbers.—Addition.—Subtraction.—Multiplication.—Cancellation.—Division..... 1

CHAPTER II

DECIMAL FRACTIONS

Reduction.—Addition.—Subtraction.—Multiplication.—Division. 14

CHAPTER III

PERCENTAGE

Base.—Rate.—Percentage.—Gain or Loss.—Commission.—Interest.—Proportions of Mixtures.—Discounts.—Efficiency..... 22

CHAPTER IV

BLUEPRINTS

Reading Blueprints.—Bevel Block.—Plain Turning.—Boring.—Taper Turning.—Assembly Drawing.—Conventions Used.... 29

CHAPTER V

MEASUREMENTS

Machinist's Rule.—Try Square.—Circle.—Calipers.—Micrometer Caliper.—Vernier Caliper..... 39

CHAPTER VI

Measurements

PAGE

Area of Rectangle.—Area of Triangle.—Area of Figures of Different Shapes.—Area of Hexagon.—Area of Circle.—Angles.—Draughtsman's Triangle.—Protractor.—Bevel Protractor.—Vernier.—Center Square 50

CHAPTER VII

Constructions

Center of Line.—Center of Arc.—Perpendiculars.—Angles.—Center of Circle.—Division of Circle 64

CHAPTER VIII

Powers. Square Root

Powers of Numbers.—Square Root.—Significant Figures.—Right-angled Triangle 72

CHAPTER IX

Lathe Work

Centering Work.—Cutting Speed.—Feed.—Calculation of Time for Work.—Taper Turning 84

CHAPTER X

Threads

Sharp "V" Thread.—United States Standard Thread.—Square Thread. — Whitworth Thread. — Acme Thread. — Worm Thread 98

CHAPTER XI

Thread Cutting

Simple Geared Lathe.—Gears for Thread Cutting.—Gear Progression.—Compound Gearing.—Tumbler Gears on Lathe.—Planer Work.—Shaper Work.—Drill Press Work 112

CHAPTER XII

Simple Machines

PAGE

Lever.—Wheel and Axle.—Pulley.—Inclined Plane.—Wedge.—Screw.—Mechanical Advantage.—Friction.—Mechanical Efficiency.—General Law of Machines.—Compound Machines.—Differential Pulley 125

CHAPTER XIII

Work. Power. Ratio

Work.—Power.—Horsepower.—Prony Brake.—Ratio and Proportion.—Gear Trains.—Pulley Trains.—Belting 143

CHAPTER XIV

Gear Calculations

Diametral Pitch.—Circular Pitch.—Depth of Tooth.—Thickness of Tooth.—Diameter of Gear Blank.—Distance between Gears. 158

CHAPTER XV

Milling Machine

Cutting Speed.—Feed.—Direct Indexing.—Simple Indexing.—Differential Indexing.—Milling Spirals 170

CHAPTER XVI

Volume and Weight

Volume of Rectangular Solid.—Weight of Square Iron Bars.—Weight of Round Iron Bars.—Weight of Sheet Iron.—Specific Gravity.—Weight of Castings 184

CHAPTER XVII

Shop Trigonometry

Functions.—Functions of Right Triangle.—Use of Trigonometric Tables.—Depth of Thread.—Spacing of Holes.—Angle of Taper 191

CHAPTER XVIII

Business Organization

PAGE

Capital. — Labor. — Accounting. — Cost to Manufacture. — The Workman.—Systems of Payment.—Pay by Day.—Piece Work.—Differential Rate of Pay.—Premium System.—Profit Sharing 202

APPENDIX

Formulas.—Decimal Equivalents of Fractions of an Inch.—Millimeter Equivalents of Inches.—Tap Drills for Machine Screw Taps.—Sizes of Tap Drills.—Screw Guage.—Emery Wheel Speeds.—Temper Colors.—Twist Drill Guage..... 211

MATHEMATICS FOR MACHINISTS

CHAPTER I

COMMON FRACTIONS

1. Fractions. An ordinary machinist's six-inch rule is

FIG. 1.

divided into *six* equal parts. Each of these equal parts is called *one-sixth* of the rule and is written $\frac{1}{6}$.

Every inch of the rule is divided into *two* equal parts, each called *one-half*, written $\frac{1}{2}$; into *four* equal parts, each

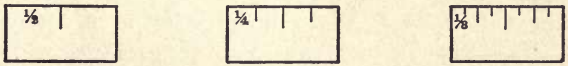


FIG. 2.

called *one-fourth*, written $\frac{1}{4}$; into *eight* equal parts, each called *one-eighth*, written $\frac{1}{8}$; and so on.

Two of the eight equal parts are called *two-eighths*, written $\frac{2}{8}$; *three* of the eight equal parts are called *three-eighths*, written $\frac{3}{8}$.

These expressions, $\frac{1}{6}$, $\frac{1}{2}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, etc., are called **common fractions.**

In a fraction such as $\frac{5}{8}$, the parts of the fraction are, the **denominator,** 8, which denominates or *gives name* to the equal parts taken and is written *below* the line; while the **numerator,** 5, numerates or *numbers* the parts taken and is written above the line.

A **proper fraction** is one whose numerator is less than its denominator, as $\frac{3}{4}$.

An **improper fraction** is one whose numerator is equal to or greater than its denominator, as $\frac{8}{8}$, $1\frac{2}{4}$.

A whole number, as 1, 2, 3, is called an **integer** to distinguish it from a fraction.

2. Reduction of Fractions to Lowest Terms. In writing a fraction it is always customary to write it in its *lowest terms.* A fraction is in its lowest terms when the numerator and denominator are the smallest integers that will express the value of the fraction.

The *value* of a fraction is not changed if both the numerator and denominator are *divided* or *multiplied* by the same number.

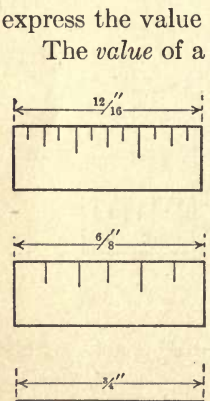


FIG. 3.

Example. Reduce $\frac{12}{16}$ to its lowest terms.

Dividing both terms of $\frac{12}{16}$ by 2 we get $\frac{6}{8}$; again dividing by 2 we get $\frac{3}{4}$, which is the fraction in its lowest terms.

$$\frac{12\div 2}{16\div 2}=\frac{6\div 2}{8\div 2}=\frac{3}{4}.$$ Answer.

We may shorten the process by dividing both terms of the fraction immediately by 4.

$$\frac{12\div 4}{16\div 4}=\frac{3}{4}.$$ Answer.

PROBLEMS

1. Reduce to lowest terms: $\frac{2}{4}$, $\frac{4}{8}$, $\frac{8}{16}$, $\frac{6}{8}$, $\frac{12}{16}$.
2. Reduce to lowest terms: $\frac{14}{16}$; $\frac{30}{32}$; $\frac{32}{64}$; $\frac{48}{64}$; $\frac{75}{100}$.
3. Reduce to lowest terms: $\frac{9}{30}$; $\frac{8}{28}$; $\frac{15}{35}$; $\frac{49}{63}$; $\frac{625}{1000}$.
4. Reduce to lowest terms: $\frac{8a}{16a}$; $\frac{14x}{16x}$; $\frac{32ab}{64ab}$; $\frac{7854d^2}{31416d^2}$.
5. How many $\frac{1}{2}$ dollars in $\frac{12}{4}$ dollars?
6. A circle is divided into 360 degrees. What part of a circle is 60 degrees?
7. In measuring the size of a hole to be bored in a steel plate, a boy found it to be $\frac{45}{64}$ of an inch. What size is this in lowest terms?
8. A thermometer in the shop reads 70° while a similar thermometer out of doors reads 35°. What fractional part of the temperature of the shop is the temperature outside?
9. Which is larger, $\frac{40}{64}$ or $\frac{9}{16}$?

3. To Change a Mixed Number to an Improper Fraction. A **mixed number** is an integer and a fraction united, as $5\frac{1}{2}$, $2\frac{7}{8}$.

Example. Change $2\frac{5}{8}$ to an improper fraction.

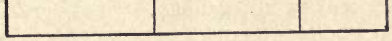


FIG. 4.

In one inch there are $\frac{8}{8}$, therefore in $2\frac{5}{8}$ inches there will be $\frac{16}{8}$, with $\frac{5}{8}$ added, which is $\frac{21}{8}$.

$$\begin{array}{r} 2\frac{5}{8} \\ 8 \\ \hline 16 \\ 5 \\ \hline 21 \end{array}$$

$\frac{21}{8}$. Answer.

PROBLEMS

1. How many 4ths of an inch in $1\frac{1}{4}$ inches? $5\frac{1}{4}$ ins.? $3\frac{3}{4}$ ins.?
2. How many 5ths in $2\frac{1}{5}$? $8\frac{3}{5}$? $10\frac{2}{5}$? $15\frac{4}{5}$?
3. Change the following to improper fractions: $3\frac{9}{16}$; $5\frac{7}{16}$; $6\frac{3}{16}$; $8\frac{13}{16}$.
4. Change to improper fractions: $64\frac{11}{15}$; $378\frac{5}{12}$.
5. How many $\frac{1}{4}$ths of a dollar in $\$2\frac{3}{4}$; $\$6\frac{1}{4}$; $\$5\frac{2}{4}$?
6. How many 10ths of a dollar in $\$3\frac{3}{10}$; $\$7\frac{5}{10}$; $\$9\frac{9}{10}$?
7. How many 8ths of an inch in a brass rod $10\frac{3}{8}$ inches long? In a rod $7\frac{7}{8}$ inches long?
8. Which is larger $2\frac{9}{16}$ or $\frac{82}{32}$?

4. To Change an Improper Fraction to an Integer or a Mixed Number. Example. Change $\frac{13}{4}$ to a mixed number.

Since there are $\frac{4}{4}$ in one inch, there will be as many whole inches in $\frac{13}{4}$ as 4 is contained in 13, which is 3 times, with $\frac{1}{4}$ left over.

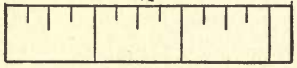


FIG. 5.

$$\frac{13}{4}=3\frac{1}{4}.$$ Answer.

PROBLEMS

1. Change to whole or mixed numbers: $\frac{47}{8}$; $\frac{121}{10}$; $\frac{25}{4}$; $\frac{18}{6}$; $\frac{872}{25}$.
2. Change to whole or mixed numbers: $\frac{636}{32}$; $\frac{1128}{64}$; $\frac{265}{15}$; $\frac{31416}{7854}$.
3. How many gallons in $\frac{9}{3}$ gallons? In $\frac{625}{8}$ gallons?
4. Change to whole or mixed numbers: $\$\frac{175}{25}$; $\$\frac{250}{50}$; $\$\frac{135}{25}$.
5. Change to whole or mixed numbers: $\frac{125a}{5a}$; $\frac{12x}{3x}$; $\frac{250ab}{25ab}$.

5. Addition of Fractions. Fractions have a *common denominator* when their denominators are equal, as $\frac{5}{8}$, $\frac{7}{8}$, $\frac{4}{8}$, $\frac{12}{8}$.

Example. Add $\frac{1}{2}$ and $\frac{3}{4}$.

Since there are $\frac{2}{4}$ in $\frac{1}{2}$ an inch, there will be 2+3 or $\frac{5}{4}$ in all.

$$\frac{1}{2}+\frac{3}{4}=\frac{2}{4}+\frac{3}{4}=\frac{2+3}{4}=\frac{5}{4}=1\frac{1}{4}.$$ Answer.

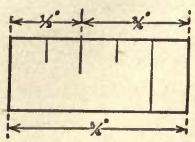


FIG. 6.

6. To find the Lowest Common Denominator. When the lowest common denominator is not readily seen it may be determined by the following method:

Example. Add: $\frac{1}{2}+\frac{3}{4}+\frac{7}{35}+\frac{3}{5}+\frac{5}{14}$.

2)	2	4	35	5	14
2)	1	2	35	5	7
5)	1	1	35	5	7
7)	1	1	7	1	7
	1	1	1	1	1

Divide by any number that will go evenly in one or more of the denominators, and bring down all numbers that will not divide evenly. Continue this process until all the numbers are divided. The lowest common denominator is the product of the divisors.

Lowest common denominator:

$2\times2\times5\times7=140$.

The fractions may now be added thus:

$$\frac{1}{2}+\frac{3}{4}+\frac{7}{35}+\frac{3}{5}+\frac{5}{14}=\frac{70}{140}+\frac{105}{140}+\frac{28}{140}+\frac{84}{140}+\frac{50}{140}=\frac{337}{140}=2\tfrac{57}{140}.$$

Answer.

PROBLEMS

1. Find the sum of: $\frac{11}{16}$; $\frac{13}{16}$; $\frac{9}{16}$; $\frac{5}{16}$.
2. Add: $\frac{45}{360}$; $\frac{30}{360}$; $\frac{60}{360}$; $\frac{90}{360}$.

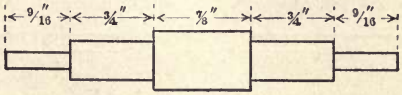


FIG. 7.

3. Find the sum of: $\frac{3}{5}$; $\frac{7}{10}$; $\frac{13}{15}$; $\frac{1}{5}$.
4. Add: $\frac{7}{8}+\frac{5}{12}+\frac{7}{16}+\frac{3}{4}$.
5. Find the sum of: $\frac{5}{6}ab$; $\frac{9}{10}ab$; $\frac{7}{12}ab$.
6. Find the sum of: $\frac{2}{3}xy$; $\frac{5}{9}xy$; $\frac{14}{18}xy$; $\frac{32}{36}xy$.
7. Find the total length of the piece shown in Fig. 7.

8. Find the total length from the outside of the first hole to the outside of the last hole in Fig. 8.

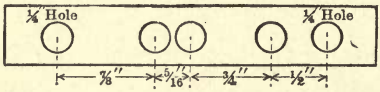


FIG. 8.

7. Addition of Mixed Numbers. Example. Find the sum of: $12\frac{1}{8}$, $9\frac{3}{4}$, $8\frac{1}{16}$, $28\frac{1}{2}$.

$12\frac{1}{8}$	$\frac{2}{16}$
$9\frac{3}{4}$	$\frac{12}{16}$
$8\frac{1}{16}$	$\frac{1}{16}$
$28\frac{1}{2}$	$\frac{8}{16}$
$58\frac{7}{16}$	$1\frac{7}{16}$

Ans.

It is convenient to add the integers and fractions separately. Change the fractions to a common denominator and add.

$$\frac{2}{16}+\frac{12}{16}+\frac{1}{16}+\frac{8}{16}=\frac{23}{16}=1\frac{7}{16}.$$

Add the 1 with the integers and write the $\frac{7}{16}$ under the fractions.

PROBLEMS

1. Find the sum of: $6\frac{1}{3}$; $5\frac{3}{4}$; $7\frac{1}{6}$; $3\frac{5}{12}$.
2. Find the sum of: $39\frac{1}{3}$; $56\frac{3}{4}$; $104\frac{5}{12}$; $88\frac{2}{3}$.

FIG. 9.

3. Add: $12\frac{1}{2}+8\frac{2}{3}+9\frac{7}{16}+225\frac{31}{32}$.
4. A man bought four kegs of nails containing respectively $100\frac{1}{2}$ lbs., $105\frac{3}{8}$ lbs., $102\frac{3}{4}$ lbs., $99\frac{7}{8}$ lbs. How many pounds did he buy?
5. What is the total length of a belt made up of the small pieces shown in Fig. 9?

8. Subtraction of Fractions. Example. From $\frac{5}{8}$ take $\frac{7}{32}$.

Since $\frac{5}{8}=\frac{20}{32}$, we have $\frac{20}{32}$ less $\frac{7}{32}$, which is $\frac{13}{32}$.

$$\frac{5}{8}-\frac{7}{32}=\frac{20}{32}-\frac{7}{32}=\frac{20-7}{32}=\frac{13}{32}$$ Ans.

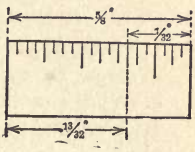


FIG. 10.

PROBLEMS

1. From $\frac{7}{8}$ take $\frac{5}{8}$.
2. From $\frac{7}{16}$ take $\frac{3}{16}$.
3. Subtract the following: $\frac{40}{64}-\frac{6}{64}-\frac{9}{64}-\frac{7}{64}-\frac{10}{64}$.
4. Find the value of: $\frac{125}{128}-\frac{13}{64}$; $\frac{22}{32}-\frac{1}{16}$.
5. How much is: $\frac{25}{64}+\frac{3}{64}-\frac{11}{32}-\frac{3}{32}$.
6. Find the value of: $(\frac{2}{3}+\frac{5}{7})-(\frac{3}{6}+\frac{5}{7})$.
7. By how much is $\frac{2}{5}-\frac{3}{7}+\frac{2}{10}-\frac{2}{15}$ greater than $\frac{15}{420}$?

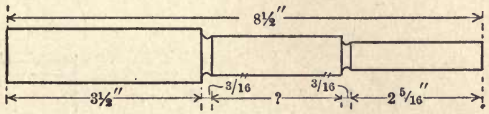


FIG. 11.

8. What number added to $\frac{3}{4}$ will make $\frac{31}{32}$?
9. Find the missing dimension in Fig. 11.
10. A man spends $\frac{3}{10}$ of his wages for board, $\frac{1}{5}$ for room rent, $\frac{1}{8}$ for carfare, $\frac{11}{40}$ for clothing and incidentals, and puts the remainder in the bank as savings. What part of his wages does he save?

9. Subtraction of Mixed Numbers. Example. From $32\frac{2}{5}$ take $27\frac{3}{4}$.

$32\frac{2}{5}$ | $\frac{8}{20}$ $(\frac{28}{20})$
$27\frac{3}{4}$ | $\frac{15}{20}$

$4\frac{13}{20}$ Ans.

We may subtract the fractions and integers separately. $\frac{2}{5}=\frac{8}{20}$ and $\frac{3}{4}=\frac{15}{20}$. Since $\frac{8}{20}$ is smaller than $\frac{15}{20}$, we take $1(\frac{20}{20})$ from the 32 and add it to $\frac{8}{20}$, making $\frac{28}{20}$. Then $\frac{28}{20}-\frac{15}{20}=\frac{13}{20}$.

PROBLEMS

1. From $\$1\frac{1}{2}$ take $\$\frac{3}{4}$.
2. From $5\frac{7}{12}$ take $2\frac{1}{6}$.
3. What number added to $6\frac{3}{4}$ will make $16\frac{2}{3}$?
4. A can contains $4\frac{7}{8}$ gallons of oil. If $1\frac{1}{2}$ gallons are poured out, how many gallons remain in the can?
5. While taking an inventory of $1\frac{1}{2}$ in. couplings, the stock boy finds that there are $5\frac{1}{4}$ dozen on hand. If $2\frac{1}{3}$ dozen are used during the week, what should be the inventory at the end of the week?

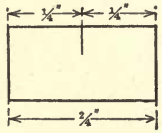


FIG. 12.

10. Multiplication of Fractions. 1. If $\frac{1}{4}$ of an inch is multiplied by 2, we have $\frac{2}{4}$ of an inch.

$\frac{1}{4}\times 2=\frac{2}{4}$. Ans.

2. If we take $\frac{1}{4}$ of an inch on each of two sides of a scale, we have the same distance on the two sides as if we had taken *two* $\frac{1}{4}$ ins. on one side of the scale, that is, $\frac{1}{4}$ of $2=\frac{2}{4}$ of 1.

("Of" means times.)

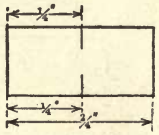


FIG. 13.

$\frac{1}{4}$ of $2=\frac{2}{4}$ of 1.
Therefore,
$\frac{1}{4}\times 2=2\times\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$.
Ans.

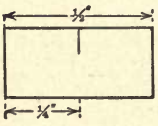


FIG. 14.

3. On the scale we see that $\frac{1}{2}$ of $\frac{1}{2}=\frac{1}{4}$.

4. In the same way $\frac{1}{2}$ of $\frac{1}{4}=\frac{1}{8}$; $\frac{1}{3}$ of $\frac{1}{5}=\frac{1}{15}$; $\frac{2}{3}$ of $\frac{1}{5}=\frac{2}{15}$.

Example. What is $\frac{2}{3}\times\frac{4}{5}$?

This means to take $\frac{2}{3}$ of $\frac{4}{5}$. Since $\frac{1}{3}$ of $\frac{4}{5}=\frac{4}{15}$, $\frac{2}{3}$ of $\frac{4}{5}=\frac{8}{15}$. Ans.

11. Cancellation. The process of multiplying fractions may often be shortened by *cancellation*, which is the striking out of *equal factors* from the numerator and denominator.

Example. $\frac{\cancel{3}}{\cancel{10}_{2}} \times \frac{\cancel{5}}{\cancel{3}} \times \frac{7}{8} = \frac{7}{16}$. Ans.

PROBLEMS

1. What is $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{2}{3}$ of $\frac{3}{4}$?
2. What is $\frac{3}{8}$ of $\frac{5}{9}$? $\frac{5}{16}$ of $\frac{3}{32}$? $\frac{7}{8}$ of $\frac{5}{12}$?
3. Multiply: $\frac{5}{6} \times 4$; $\frac{3}{8} \times 6$; $\frac{5}{7} \times \frac{3}{4}$; $\frac{2}{3} \times \frac{5}{8}$.
4. Multiply: $\frac{48}{56} \times \frac{25}{35}$.
5. What is $\frac{34}{65} \times \frac{11}{17}$?
6. How much is $\frac{14}{10} \times \frac{76}{49}$?
7. Show that $\frac{3}{4} \times \frac{5}{6} = \frac{5}{6}$ of $\frac{3}{4}$.
8. What is $\frac{2}{3}$ of $\frac{6}{9}$ of $\frac{8}{12}$?
9. Multiply: $\frac{1}{2} \times \frac{4}{5} \times \frac{15}{32} \times \frac{8}{45}$.
10. A boy works 5 hours for $\frac{1}{8}$ of a dollar an hour. What part of a dollar does he receive in wages?

12. Multiplication of Mixed Numbers. Example. Multiply $3\frac{1}{5}$ by $1\frac{1}{2}$.

It is generally easy to reduce the mixed numbers to fractions before multiplying, thus:

$$3\tfrac{1}{5} \times 1\tfrac{1}{2} = \frac{\overset{8}{\cancel{16}}}{5} \times \frac{3}{\cancel{2}_{1}} = \frac{24}{5} = 4\tfrac{4}{5}.$$ Ans.

PROBLEMS

1. Multiply $2\frac{1}{2}$ by 5; $3\frac{1}{4}$ by $5\frac{1}{8}$.
2. What will $12\frac{1}{2}$ lbs. of white lead cost at $8\frac{3}{4}$ cents a pound?
3. What is the freight charge on $5\frac{1}{4}$ tons of scrap iron at $\$1\frac{3}{10}$ per ton?
4. A boy works 8 hours and 15 minutes for $16\frac{2}{3}$ cents per hour. How much does he receive?

5. If a train goes $35\frac{2}{3}$ miles an hour, how far will it go in $3\frac{3}{4}$ hours at the same rate of travel?

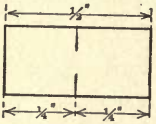


FIG. 15.

$\frac{1}{2} \div 2 = \frac{1}{4}$. Ans.

or $\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Ans.

13. Division of Fractions. Examples. 1. Divide $\frac{1}{2}$ by 2. If $\frac{1}{2}$ is divided into two equal parts each of these parts must be $\frac{1}{4}$.

Dividing by 2 is the same as multiplying by $\frac{1}{2}$.

2. How many times is $\frac{1}{2}$ an inch contained in 2 ins.? 4 times.

Dividing by $\frac{1}{2}$ is the same as multiplying by 2.

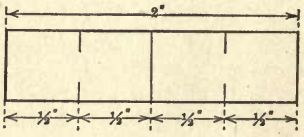


FIG. 16.

$2 \div \frac{1}{2} = 2 \times \frac{2}{1} = 4$. Ans.

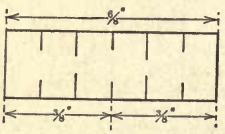


FIG. 17.

$\frac{6}{8} \div \frac{3}{8} = \frac{6}{8} \times \frac{8}{3} = \frac{6}{3} = 2$. Ans.

3. How many times is $\frac{3}{8}$ contained in $\frac{6}{8}$? 2 times.

Dividing by $\frac{3}{8}$ is the same as multiplying by $\frac{8}{3}$.

4. How much is $\frac{7}{8} \div \frac{3}{5}$?

Dividing by $\frac{3}{5}$ is the same as multiplying by $\frac{5}{3}$. $\frac{7}{8} \div \frac{3}{5} = \frac{7}{8} \times \frac{5}{3} = \frac{35}{24} = 1\frac{11}{24}$. Ans.

PROBLEMS

1. If a man can do $\frac{8}{9}$ of a piece of work in 8 days, how much can he do in 1 day? In 2 days?

2. How much is $\frac{3}{8}$ of an inch divided by 2? By 4?

3. Divide: $\frac{16}{21}$ by 8; $\frac{33}{40}$ by 11; $\frac{60}{91}$ by 20.

4. Why does multiplying the denominator of $\frac{16}{19}$ by 4, divide the fraction by 4?

5. Find the value of: $\frac{3}{16} \times \frac{8}{12} \div \frac{7}{8}$.

6. Find the value of: $\frac{1}{31} \frac{1}{41} \frac{1}{16} \times \frac{7}{2} \frac{8}{8} \frac{5}{4} \div \frac{1}{8}$.

7. A man makes 88 pieces of work in $5\frac{1}{2}$ hours. How many pieces does he make in an hour?

8. A moving picture camera takes 5 feet of film in 10 turns of the operating crank. How many feet of film does it take in 2 turns of the crank? In 3 turns of the crank?

14. Division of Mixed Numbers. Example. Divide $6\frac{7}{8}$ by $2\frac{1}{2}$.

It is generally easy to first reduce the mixed numbers to fractions and then perform the division.

$$6\tfrac{7}{8} \div 2\tfrac{1}{2} = \tfrac{5}{8}\tfrac{5}{} \div \tfrac{5}{2} =$$

$$\frac{\overset{11}{\cancel{55}}}{\underset{4}{\cancel{8}}} \times \frac{2}{\underset{1}{\cancel{5}}} = \frac{11}{4} = 2\tfrac{3}{4}. \quad \text{Ans.}$$

PROBLEMS

1. Divide $16\frac{2}{3}$ by $33\frac{1}{3}$.

2. Divide $6\frac{1}{4}$ by $1\frac{1}{8}$.

3. Find the value of: $\dfrac{8\frac{4}{9}}{12\frac{2}{3}}$.

4. Find the value of: $\dfrac{\frac{3}{5} \times \frac{2}{6} \times \frac{7}{8} \times \frac{5}{4}}{\frac{14}{6} \times \frac{3}{16} \times \frac{1}{4} \times 2}$.

5. If a man travels $2\frac{2}{3}$ miles in 1 hour and 15 minutes, how far does he travel in 2 hours?

REVIEW PROBLEMS

1. The weights of several castings are $520\frac{1}{2}$ lbs., $365\frac{1}{4}$ lbs., and 1100 lbs., respectively. Find the total weight.

2. How much space would 6 lathes occupy placed end to end, if each lathe is $8\frac{1}{2}$ feet long, and a space of $3\frac{1}{2}$ feet is left between lathes?

3. Four bolts are required, each $2\frac{3}{4}$ inches long. Allowing $\frac{7}{8}''$ for cutting and finishing, how long a piece of stock will be needed?

4. The distance between two surfaces was measured by standard distance blocks. The following blocks were used: $\frac{5}{8}$, $\frac{1}{4}$, $\frac{9}{16}$, $\frac{7}{32}$, $\frac{5}{64}$. What was the total distance?

5. The base of a machine weighs 350 lbs., which is $\frac{7}{10}$ of its total weight. What is the total weight?

Solution. Suppose we consider the whole machine as made up of ten equal parts. 7 of those equal parts weigh 350 lbs., therefore 1 of the equal parts would be $\frac{1}{7}$ of 350 lbs., or 50 lbs. But 1 equal part is $\frac{1}{10}$ of the machine, therefore, the whole machine or $\frac{10}{10}$ would weigh 10 times 50 lbs. or 500 lbs. Ans.

6. A factory discharged 40 men, which was $\frac{2}{5}$ of the whole number employed in the factory. How many men were employed?

7. Arrange in order of magnitude: $\frac{15}{16}$, $\frac{7}{8}$, $\frac{31}{32}$, $\frac{3}{4}$, $\frac{5}{8}$.

8. What is the total length of iron pipe required for a railing

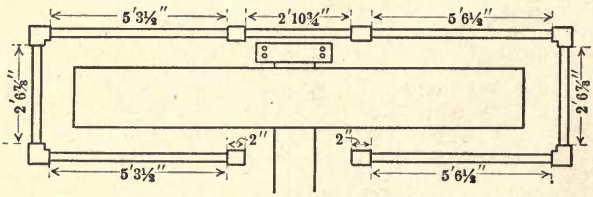


FIG. 18.

around the wheel shown in Fig. 18, allowing 1 inch at each side of a coupling and corner?

9. Find the difference between: $\frac{1}{2}+\frac{1}{6}+\frac{1}{4}+\frac{1}{8}$ and $\frac{2}{3}-\frac{1}{3}+\frac{1}{5}-\frac{2}{5}$.

10. Time cards for a certain piece of work show 1 hour 15 minutes lathe work, 2 hours 45 minutes shaper work, and 3 hours 40 minutes bench work. What is the cost at \$2.50 per day of 8 hours?

11. It takes $8\frac{1}{2}$ hours of a man's time to complete a certain piece of work. If $2\frac{1}{3}$ hours are spent in turning, $2\frac{1}{12}$ hours in finishing, and the rest of the time in assembling, how many hours are spent in assembling?

12. Which is cheaper, to have a boy at $12\frac{1}{2}$ cents an hour work $8\frac{1}{2}$ hours on a job, or to hire a man at $16\frac{2}{3}$ cents an hour, who can do the job in 6 hours?

13. A cylindrical piece of work is turned in a lathe. If the

diameter at the start is $4\frac{1}{8}$ inches and 3 cuts are taken, the first $\frac{1}{16}$ deep, the second $\frac{3}{32}$, and the last $\frac{1}{64}$, what is the finished diam.?

14. A forging weighs $62\frac{1}{2}$ lbs. in the rough. When finished it weighs $37\frac{3}{4}$ lbs. If the forging cost 18 cents a pound and the metal turned off can be sold for $2\frac{1}{8}$ cents a pound, what is the net cost of the material?

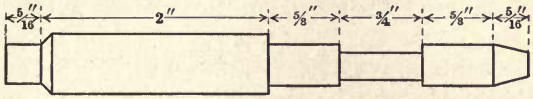


FIG. 19.

15. Find the total length of the piece of work shown in the sketch Fig. 19.

16. A man works overtime at "time and a half" for 3 hours and 20 minutes. If he gets $2.60 per day of 8 hours, how much does he get for overtime?

17. A brass rod 3 ft. 8 in. long is to be cut into 5-inch pieces. If there is a waste of $\frac{1}{16}$ inch in cutting each piece, how many pieces can be made?

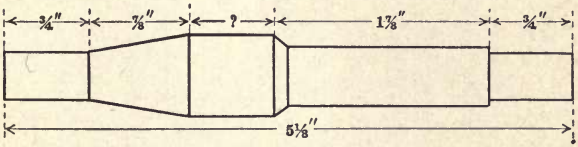


FIG. 20.

18. Find the length of the missing dimension in the sketch Fig. 20.

19. An iron bar weighs $2\frac{5}{10}$ lbs. to a foot. How long a piece must be taken to weigh 5 lbs.?

20. A piece of work can be made of steel or cast iron. If made of steel it will take $7\frac{3}{4}$ lbs. at $6\frac{1}{8}$ cents a pound; if made of cast iron it will take 14 lbs. at $2\frac{7}{8}$ cents a pound. Which material would be cheaper?

CHAPTER II

DECIMAL FRACTIONS

15. Decimal Fractions. A **decimal fraction** is one whose denominator is 10 or some product of 10, as 100, 1000, etc.

Decimal fractions may be written in two ways, as $\frac{3}{10}$ or .3; $\frac{25}{100}$ or .25; $\frac{625}{1000}$ or .625.

The *table of order* of integers and decimals for ordinary use is as follows:

Ten-thousands.	Thousands.	Hundreds.	Tens.	Units.	**Decimal Point.**	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.
0	0	0	0	0	·	0	0	0	0	0

The decimal point is a period placed at the *left* of the order of *tenths*, to show decimal *orders*.

16.(*a*) **To Reduce a Decimal Fraction to a Common Fraction. Example.** Reduce .625 to a common fraction.

$$.625=\frac{625}{1000}=\frac{25}{40}=\frac{5}{8}. \quad \text{Ans.}$$

PROBLEMS

1. Change to common fractions: .5; .25; .75; .875.
2. Change to common fractions: .1875; .5625; .9375; .4375.
3. The size of a drill is given as .8125. What size is this in 16ths of an inch?
4. What fractional part of a dollar is $87\frac{1}{2}$ cents? $37\frac{1}{2}$ cents? $62\frac{1}{2}$ cents?
5. The size of two milling machine arbor washers is given as .6875 in. and .3125 in. respectively. Change these sizes to common fractions and add the fractions.

16.(*b*) To Reduce a Common Fraction to a Decimal Fraction. Example. Change $\frac{3}{4}$ to a decimal fraction.

First Method:

$$\frac{3}{4}\times\frac{25}{25}=\frac{75}{100}=.75.$$

Ans.

Second Method:

$$4\overline{)3.00}\quad .75 \text{ Ans.}$$

2 8

20

20

1. Multiplying both terms of the fraction by some number that will make the denominator a product of *ten* we have $\frac{75}{100}$ or .75. Answer.

2. Since numbers are written by orders that differ from each other by *tens*, when we divide one number by another we are really dealing with *tens;* we may, therefore, change a common fraction to a decimal by simply dividing the numerator by the denominator.

PROBLEMS

1. Change to decimal fractions: $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$.
2. Change to decimal fractions: $\frac{3}{16}$; $\frac{5}{16}$; $\frac{7}{16}$; $\frac{9}{16}$.
3. How many hundredths are there in $\frac{1}{20}$; $\frac{3}{20}$; $\frac{5}{20}$; $\frac{11}{20}$?
4. Change to decimal fractions: $\frac{25}{32}$; $\frac{3}{40}$; $\frac{13}{40}$; $\frac{80}{125}$.
5. How many cents in: $\$\frac{2}{5}$; $\$\frac{1}{5}$; $\$\frac{3}{8}$; $\$\frac{1}{6}$?

6. Sheet iron of No. 26 gauge is $\frac{3}{160}$ of an inch thick. Express this size decimally.

17. Addition of Decimals. Example. Add the following decimals.

15.84
37.08
81.206
4.7
3.6254

142.4514 Ans.

Decimals may be added like integers by simply writing them so that figures of the same *order* stand in the same columns.

PROBLEMS

1. Add: 6.48; 2.7; 8.634; 4.532.
2. Add: 18.06; 4.15; 2.45; 12.785.
3. Find the sum of: 625 thousandths, 250 thousandths, 8 ten-thousandths, 75 hundredths, 4 tenths.
4. Find the total cost of the following:

1 micrometer caliper	$5.50
1 depth gauge	1.25
1 automatic centre punch	1.25
1 pair dividers	1.40
1 steel rule	.65
1 center gauge	.35
1 try square	3.50

5. Five washers of the following sizes are placed on the arbor of a milling machine: .75, .4375, .375, .5, .3125. Find the length over all.
6. Standard distance blocks are used to measure the opening in a piece of work. If the following were used, what was the size of the opening? .0625, .1875, .8125, .6875.
7. A screw with U. S. standard thread measures .6201 in. at the bottom or root of the thread. If the thread on each side is .06495 in. deep, what is the outside diameter of the screw?

18. Subtraction of Decimals. Example. From 28.345 take 15.04.

28.345
16.04
12.305 Ans.

Write the figures so that they stand in columns of the same *order*, and subtract as in the case of integers.

PROBLEMS

1. From 38.256 take 27.8.
2. From .063 take .005.
3. From 256 thousandths take 25 hundredths.
4. A barometer read 29.325 ins. at the top of a mountain and 30.152 ins. at the bottom of the mountain. Find the difference in readings.
5. A boiler tube measured 3.07 ins. outside diameter and 2.945 ins. inside diameter. Find the thickness of the tube.
6. A tapering piece of work measured 1.25 ins. in diameter at one end and .875 in. in diameter at the other. Find the difference in diameter.
7. A piece of work 1.4375 ins. in diameter is placed in a lathe and a cut .0625 in. deep is taken on it. What is the diameter of the finished piece?

19. Multiplication of Decimals. Example. Multiply .25 by .5.

.25 (two places)
.5 (one place)
.125 (three places) Ans.

Since $.25=\frac{25}{100}$ and $.5=\frac{5}{10}$ the product is:

$$\frac{25}{100}\times\frac{5}{10}=\frac{125}{1000}=.125 \quad \text{Answer.}$$

The number of decimal places in the product is always equal to the *sum* of the decimal places in the factors; in this case three decimal places.

Example. Multiply .0432 by 3.8.

.0432 (four places)
3.8 (one place)

In this case we point off five decimal places.

3456
1296

.16416 (five places)

PROBLEMS

1. Multiply .625 by .75.
2. Multiply 64 by .032.
3. Multiply .0056 by .005.
4. Multiply 2.25 by 40.
5. Multiply sixteen hundred by sixteen hundredths.
6. A man works for 4.5 days at the rate of $2.75 per day. How much does he receive?
7. A milling cutter turns 50 revolutions with a feed of .02 in. per revolution. What is the distance cut?
8. A planer has a feed of .03 in. per stroke. What is the width cut in 100 strokes?

20. Division of Decimals. 1. **Example.** Divide .288 by .036.

Since $.288 \div .036 = \frac{288}{1000} \div \frac{36}{1000}$, we have

.036).288(8. Ans.
288

$$\frac{\cancel{288}}{\cancel{1000}} \times \frac{\cancel{1000}}{\cancel{36}} = \frac{8}{1} = 8. \quad \text{Ans.}$$

2. **Example.** Divide .875 by .25.

.25).875(3.5 Ans.
75

125
125

Since $.875 \div .25 = \frac{875}{1000} \div \frac{25}{100}$ we have

$$\frac{\cancel{875}^{35}}{\cancel{1000}_{10}} \times \frac{\cancel{100}}{\cancel{25}} = \frac{35}{10} = 3.5. \quad \text{Ans.}$$

3. **Example.** Divide .0078 by 2.6.

Since $.0078 \div 2.6 = \frac{78}{10000} \div 2\frac{6}{10}$, we have

$$\frac{78}{10000} \div \frac{26}{10} = \frac{\overset{3}{\cancel{78}}}{\cancel{10000}_{1000}} \times \frac{\cancel{10}}{\cancel{26}} = \frac{3}{1000} =$$

.003 Ans.

2.6).0078(.003 Ans.
78

From the above illustrations we see that division of decimals is the same as with ordinary numbers with the exception of placing the decimal point in the answer. In every case there are as many decimal places in the answer as the number of decimal places in the number divided is *greater than* the number of decimal places in the divisor.

When the number of decimal places in the number divided is less than the number of decimal places in the divisor, we must make them equal to the number in the divisor. In this case the answer is a whole number.

PROBLEMS

1. Divide 2.56 by 64.
2. Divide .675 by 75.
3. Divide 26.75 by .25.
4. Divide 1.2625 by 2.5.
5. Divide six thousandths by six hundred.
6. At $2.80 each how many milling cutters can be bought for $14.00?
7. How many metal slitting saws each .3125 in. thick can be placed in a box 5 ins. long?
8. A man who has paid 27.5 cents an hour received $1.76. How long did he work?

REVIEW PROBLEMS

1. A cubic inch of lead weighs .41 lb. Find the weight of 1 cu.ft. (1728 cu.ins.).

2. If there are 2.54 centimeters in an inch, how many centimeters are there in a yard?

3. A taper is stated as .6 in. per foot. How much would this be per inch?

4. A cylindrical piece of work being turned in a lathe was found to be 1.6875 ins. in diameter. If a cut $\frac{1}{32}$ in. deep is taken on the work, what is the diameter when finished?

5. If a cubic foot of water weighs 62.5 lbs., how many pounds of water will there be in a tank which holds 120 cu.ft.?

6. If a taper is stated as .05 in. per inch, how much would it be per foot?

7. Write the fractions from $\frac{1}{4}$ in. to $\frac{3}{8}$ in. by 32ds, and change these fractions to decimal fractions.

8. Express $\frac{1}{64}$ larger than $\frac{3}{4}$ of an inch as a decimal.

9. A piece of work called for $\frac{7}{8}$ in. diameter. If it was turned $\frac{5}{1000}$ in. too small, what was the diameter?

10. If a casting is made up of .65 copper and .35 zinc, how many pounds of each metal would there be if the casting weighed 40 lbs.?

11. If steel bars weigh 2.67 lbs. per foot, how much must be paid for 2000 ft. at $1.90 per 100 lbs.?

12. A man who is getting $2.60 a day of 8 hours works over-

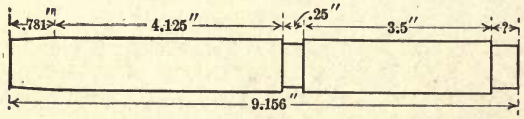


FIG. 21.

time for 3 hours at "time and a half." How much does he get for overtime?

13. Find the missing dimension in Fig. 21.

14. What is the total cost of the following:

6 milling cutters at $3.10.....
12 end mills at $2.15.....
6 screw slotting cutters at $.30.....
2 hand reamers at $2.40.....
12 steel rules at $.65.....

15. How much would be deducted from the pay of a man who gets $2.60 per day of 8 hours, for a loss of time of 2 hours and 40 minutes?

16. The diameter of No. 1 wire is given on different wire gauges as follows:

Brown and Sharpe	.2893 in.
Birmingham	.3000 in.
Washburn	.2830 in.
Imperial	.3000 in.
Stubs	.2270 in.
U. S. Standard	.2812 in.

Which two gauges differ the most in size and how much?

17. Find the length over all in Fig. 22.

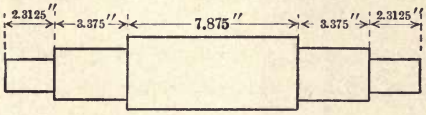


FIG. 22.

18. If a piece of steel tape expands .00024 in. for every inch of its length when heated, how much will 50 ft. expand?

19. If lead is 11.38 times as heavy as water, what will 1 cu.ft. of lead weigh, if a cubic foot of water weighs 62.5 lbs.?

20. In France distances between cities are measured in kilometers. A kilometer is .62137 mile. How many miles would there be between two places 25 kilometers apart?

CHAPTER III

PERCENTAGE

Percentage is a form of numerical reckoning where things are taken *by the hundred.* It may be regarded as a particular form of decimal fractions where the denominator is always 100; thus 5 per cent of anything means $\frac{5}{100}$ of it.

The sign % is generally used instead of the words "*per cent.*"

A **percentage** of a number is the *result* obtained by taking any per cent of that number.

The **rate per cent** or **rate** is the number of hundredths taken and may be written as 5%, $\frac{5}{100}$, or .05.

The **base** is the number on which the percentage is found.

The *base, rate and percentage* have such relations to each other that if any *two* of them are given, the other may be found.

21. To Find Percentage when Base and Rate are Given. Example. How much is 5% of 400?

400 — The *base* is 400 and the rate is 5%.
.05 — Since 5% = .05, then .05 × 400 = 20. Ans.

20.00 Ans.

Note. It must always be kept in mind that the sign % means *hundredths* and is equivalent to *two* decimal places when used; thus 5% is .05, .5% is .005, and .05% is .0005.

PROBLEMS

1. What is 8% of 350?
2. What is 3% of $37.50?
3. If there are 60 men working in a shop and 25% of them are discharged, how many men are discharged?
4. Out of a lot of 80 castings $7\frac{1}{2}\%$ (.075) were found to be defective. How many were defective?
5. A man who receives $3.00 per day has his wages raised 10%. How much will he now receive per day?

22. To Find the Rate when the Base and Percentage are Given. Example. What part of $12 is $3?

$3 is $\frac{3}{12}$ or $\frac{1}{4}$ of $12; $\frac{1}{4}=\frac{25}{100}=25\%$. Ans.

We obtain the same result if we divide the smaller number by the larger.

12)3.00(.25 or 25% Ans.
24

60
60
—

PROBLEMS

1. What per cent of 450 is 90?
2. What per cent of $240 is $72?
3. What per cent of 75 lbs. is $16\frac{1}{2}$ lbs.?
4. In turning 50 pieces of work on a lathe, a boy spoiled 4. What per cent did he spoil?
5. In transmitting 80 horse-power by belts, 1.6 horse-power was lost by slippage. What per cent was lost?

23. To Find the Base when the Percentage and Rate are Given. Example. $45 is 15% of what sum of money?

Solution. Since $45 is 15% or $\frac{15}{100}$ of a sum of money,

$\frac{1}{100}$ will be $\frac{1}{15}$ of 45 which is 3; $\frac{100}{100}$ or the whole sum will therefore be 100×3 which is 300. $300. Ans.

We obtain the same result if we divide the *percentage* by the *rate.*

.15)45.00(300. Answer.
45
—

PROBLEMS

1. 40 is 8% of what number?
2. $15 is 25% of what sum of money?
3. $\frac{5}{16}$ in. is what per cent of an inch?
4. The interest on a sum of money at 5% is $18. What is the sum of money?
5. Thirteen pounds of copper are put into a brass casting. If this is 65% of the total casting, what is the weight of the casting?

24. Applications of Percentage.

I. Gain or Loss

1. **Example.** A man who is getting $2.50 per day is given a 10% raise. How much will he get per day?

Solution. A 10% raise means that 10% of the amount he is now getting is to be added to his wages.

Ten per cent of $2.50 = $.25. He will therefore get $2.50 + $.25 = $2.75 per day. Ans.

2. **Example.** If the man in the above problem while getting $2.75 per day is reduced 10% what will he get?

Solution. A 10% reduction means 10% of what he is getting, which is 10% of $2.75 = $.27½. Therefore he will get $2.75 − $.27½ = $2.47½. Ans.

Note that the 10% reduction being taken on a larger amount as a *base* is more than the raise that he got at first.

II. Commission

Commission is an allowance paid to an agent for the transaction of business.

Example: A salesman sold \$1200 worth of goods at a commission of 5%. How much did he receive?

Solution. \$1200×.05=\$60. Ans.

III. Interest

Interest is compensation for the use of money. The amount of interest to be paid is always reckoned as a definite per cent of the amount of money borrowed.

Example. A man wishing to start a factory, borrowed \$3000 at 6% interest. How much did he pay per year for the use of the money?

Solution. \$3000×.06=\$180.00. Ans.

IV. Proportions of Mixtures

Certain substances such as brass, babbitt, bronze, etc., are made up of combinations of metals in different proportions. Since different weights of these metals are made up at any one time, it is convenient to state the amount of each metal to be put in, as a per cent of the whole.

Example. Bronze for bearings is sometimes made up of copper 77%, lead 15%, tin 8%. If 110 lbs. of bronze is to be made, how many pounds of each metal must be taken?

Solution.

Copper	110×.77=	84.7 lbs.
Lead	110×.15=	16.5 lbs.
Tin	110×.08=	8.8 lbs.
Total	=110.0 lbs.	

V. Discounts

Dealers generally publish prices of articles in catalogues. These prices are often subject to a *discount* or reduction. Sometimes there are several discounts to be taken one after the other from the catalogue price. This is done in the following manner: the first discount is taken off the full price in the catalogue; the second discount is then taken off what is left after the first discount is subtracted; the third is then taken off what is left after the second is subtracted, and so on.

Example. Morse straightway drills No. 114A, size $\frac{1}{2}$ in., length 6 ins., are quoted in a dealer's catalogue at \$8.00 per doz., with discount at 40–10–5%. What is the net cost of 1 doz. of these drills?

Solution.

\$8.00 × .40 = \$3.20 first discount.

\$8.00 − \$3.20 = \$4.80. \$4.80 × .10 = \$.48 second dis.

\$4.80 − \$0.48 = \$4.32. \$4.32 × .05 = \$.21$\frac{6}{10}$ third dis.

\$4.32 − \$.21$\frac{6}{10}$ = \$4.10$\frac{4}{10}$, net cost. Ans.

VI. Efficiency

In running machinery it is impossible to avoid wasting some energy in friction of the moving parts of the machine. We may reduce friction to a very small quantity by oil, ball-bearings, etc., but we can never do away with it entirely.

Since every machine wastes some energy it is clear that no machine can do work continuously without a supply of energy from without. Not recognizing this fact, many inventors have wasted thousands of dollars in trying to discover "perpetual motion" machines, which are clearly impossible.

Efficiency is a term applied to a machine to tell how well it does its work; i.e., with how little loss of energy. If a machine could give back all the energy put into it, that machine would be a perfect machine and would have an efficiency of 100%. We may therefore think of the efficiency of a machine as the *percentage* of useful work that can be gotten out of it.

Example. A milling machine driven by a 2 H.P. motor was found to develop 1.5 H.P. while cutting. What was the efficiency of the machine?

Solution. We find what per cent 1.5 H.P. is of 2 H.P.

$$2)1.50(.75 \text{ or } 75\% \quad \text{Ans.}$$
$$14$$
$$\overline{10}$$
$$10$$
$$\overline{}$$

REVIEW PROBLEMS

1. A shop employing 300 men adds 60 more. What is the per cent of increase?

2. A contract on work was taken at $1500. When the work was completed the actual cost was found to be $1410. What was the per cent of profit on the contract price?

3. Write 40% as a decimal fraction and as a common fraction in its lowest terms.

4. A man invests $5000 in a machine shop. At the end of the year he receives a dividend of 5% on the money invested. How much does he receive?

5. A motor runs at 10 H.P. If this power is transmitted to a line shaft with 3% loss by slipping of the belts, what is the horse-power of the line shaft?

6. Write the following as per cent of a dollar: $\$\frac{1}{8}$, $\$\frac{1}{4}$, $\$\frac{1}{2}$, $\$\frac{5}{8}$.

7. A drop forging weighed 145 lbs. before finishing and 113.1 lbs. after finishing. What per cent of metal was turned off?

8. Time cards for a piece of work showed 2 hours 30 minutes

shaper work, 3 hours 15 minutes lathe work, and 1 hour 15 minutes bench work. What per cent of the total time was spent at the shaper, bench, and lathe?

9. Babbitt metal is made up as follows: 80% tin, 10% copper, and 10% antimony. How many pounds of each metal are needed for 75 lbs. of babbitt?

10. A shop decides to raise wages 12½%. If a man is now getting $2.80 per day, and a boy is getting $1.60 per day, how much will each get after the raise?

11. Cleveland jobber's reamers No. 128A, size 1 in., are quoted at $3.70 each, with a discount of 20–7½%. What is the net cost?

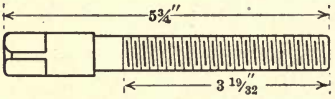


FIG. 23.

12. Through what per cent of its length is the set screw, Fig. 23, threaded?

13. The capacity of a shop is increased 10% by the introduction of new machinery. If it now turns out 275 pieces of work a day, how many pieces did it turn out before? (Suggestion: If running at 100% before, it now runs at 110%.)

14. If there are 2.54 centimeters in an inch, what per cent of an inch is 1 centimeter?

15. Write 62½% as a common fraction and reduce it to its lowest terms.

16. A machinist is paid 27½ cents an hour. He works for a full day of 8 hours and puts in 4 hours overtime at a "time and a half." What per cent of his regular day wages does he get for overtime?

17. A man pays $15.75 for 3½ weeks board. If his pay is $15 per week, what per cent of his wages does he pay for board?

18. Morse twist drills in sets, No. 5 straight shank, $\frac{1}{16}$ in. to ½ in. by 64ths, mounted, are quoted at $10.00 per set, with a discount of 55–5%. What is the net cost of a set of drills?

19. A job shows a net profit of $168. If this is 8% of the contract price, what is the contract price?

20. A brass casting contains 65% copper and 35% zinc. If the whole casting weighs 76 lbs., how many pounds of copper and how many pounds of zinc are in it?

CHAPTER IV

BLUEPRINTS

If a machinist is to do his work intelligently, he must thoroughly understand what he is to do before he begins his work. The easiest way of giving this information in the shop is by means of blueprint drawings or sketches. This method frequently fails to give the desired information because the average machinist does not understand how to get at the meaning of the blueprint which is given to him. There are certain definite ways of expressing things in blueprints, and it is the aim of this chapter to bring out some of the common ways in which those things are expressed.

25. Chipping Bevel on Cast-iron Block. In the sketches, Fig. 24, we have two different views of the work shown. The first sketch is a plan or top view, as the work would appear to one standing above and looking down on the top of it.

Heavy lines, ————————, are always used to show the outside or visible parts of a piece of work.

Dimension and Termination Lines |←——— ———→| or ———→| |←——— are used to indicate distances. In this case the dimension lines indicate that the cast-iron block is 4 ins. long and 2 ins. wide.

The second sketch is an elevation or side view, as the block would appear to one standing at the side and looking at the block on a level with the eye. If only the

plan view were given, there might be some confusion in regard to the inside lines, which indicate the bevel as it will look when finished. This side view shows clearly how the bevel will look at the ends of the block and leaves no doubt about how the bevel will slope.

The side view shows that the height of the block is

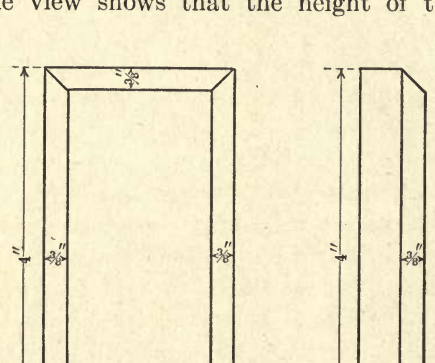


FIG. 24.

1 in., which could not be shown on the plan view. Since all parts of the finished block are visible in both sketches, heavy lines are used throughout.

26. Plain Turning; Cast Iron. The three drawings of Fig. 25 represent respectively: the stock; the work centered, ends faced, and roughed out; finished work.

Sketch 1. As before we see that heavy lines are used to represent the visible parts of the piece of work. Dimension lines give the total length of the rough stock as $8\frac{5}{8}$ ins. and the diameter as $1\frac{3}{8}$ ins. The circle drawn at the

center shows what would be seen if we should cut across the stock at that point and look at the cut end. This is called a **cross-section.** The small diagonal lines drawn at the cross-section are a conventional way of showing that the stock is *cast iron.* Similar lines on drawings always indicate that the material is cast iron. The line going through the middle of the sketch is the *center line* of the work.

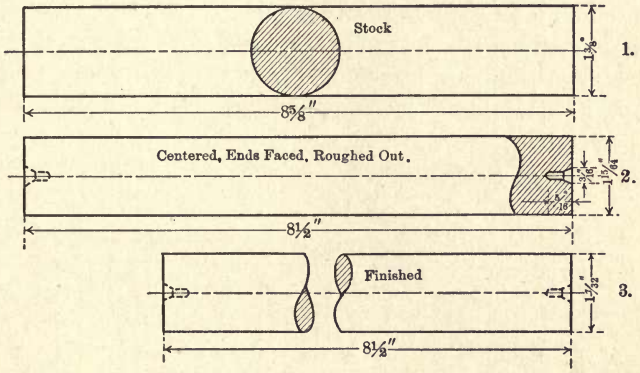


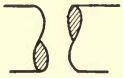
Fig. 25.—Plain Turning.

Center lines — — · — — — — — — are always made with long ($\frac{1}{2}$ in.) and short ($\frac{1}{8}$ in.) dashes.

Sketch 2. In sketch 2, the dimension lines indicate that the piece of work has been finished to the proper length, $8\frac{1}{2}$ ins. This should always be done first, as *the ends must be faced before turning.* A roughing cut has been taken over the piece, making the diameter $1\frac{15}{64}$ ins. The center on which the work is turned is not visible in this sketch at the left-hand side, and is therefore shown by lines of *short dashes.* It is customary to make the

diameter of the center hole about $\frac{1}{7}$ the diameter of the finished piece.

Construction lines - - - - - - - - - - - - - - are always shown by short ($\frac{1}{8}$ in.) dashes.

Sketch 3. This shows the work when finished. The  mark at the center means that there was not room enough on the blueprint to show the full length of the piece according to the scale used, so the middle of the sketch has been dropped out to make room. Notice that although the middle of the piece has been dropped out, the length is *stated as full.*

27. Boring a Bush Plate. In the bush plate, Fig. 26,

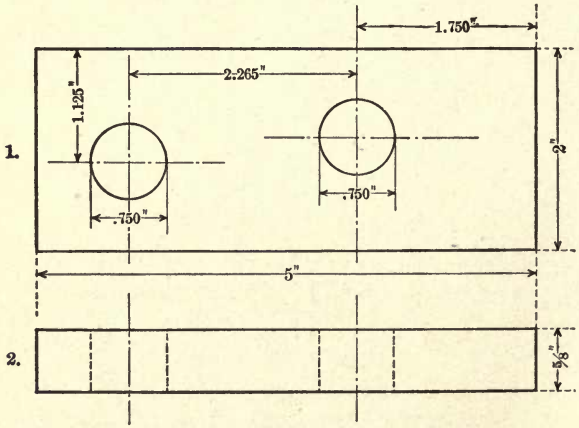


FIG. 26.—Bush Plate.

very accurate spacing of the holes is desired, so the dimensions are given in thousandths of inches. In sketch 1 two center lines are given, crossing at the exact center of each

hole. In sketch 2, construction lines show the position of the holes.

28. Taper Turning and Boring. In the sketch, Fig. 27, the *outside taper* is stated as .6 in. per foot. This means that

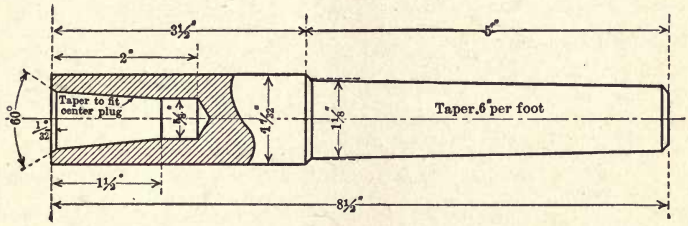


FIG. 27.—Taper Turning and Boring.

if the taper extended 1 ft. along the piece of work one end of the work would be .6 in. smaller than the other end. The taper per inch along the piece of work would be only $\frac{1}{12}$ as much as it would be per foot, or $\frac{6}{10} \times \frac{1}{12} = \frac{1}{20}$ in.

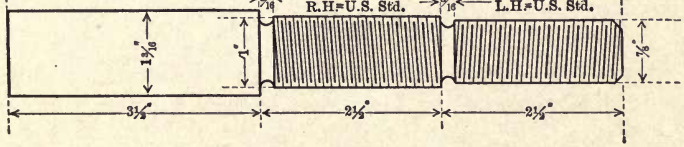


FIG. 28.—Turning and Threading.

per inch. The dimension given at the extreme large end of the taper is shown by the dotted lines to be $1\frac{1}{8}$ in. Five inches away, or at the end of the piece, the work would be $\frac{5}{20}$ in. smaller than $1\frac{1}{8}$ in. or $\frac{7}{8}$ in. The angle of the

bevel at this end of the piece is not given. When not given, it is customary to make this bevel 60 degrees.

The shoulder at the middle of the piece of work is left just as the tool finishes it in turning.

An *inside taper* is shown at the left-hand end of the piece of work. The dimension $\frac{5}{8}$ in. given at the bottom of the hole means that a hole is first to be bored in the piece $\frac{5}{8}$ in. in diameter. This inside taper is not stated on the drawing, but is made to fit a standard gauge or plug. The taper is usually marked on the plug.

The 60° mark at the end of the drawing means that a 60-degree countersink is to be made at that end of the work. This countersink is shown to be $\frac{1}{32}$ in. deep.

29. Thread Cutting. *Threads* are often represented as in Fig. 29.

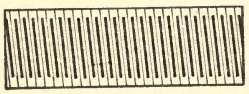


FIG. 29.

In Fig. 28, R. H. U. S. Std., means that a right-hand United States Standard Thread is to be cut. The outside diameter of this thread is shown as 1 in. From a table of standard threads we may learn that a United States Standard Thread 1 in. outside diameter has 8 threads per inch.

Necking $\frac{3}{16}$ in. wide and deep enough to reach the bottom of the thread is provided so that the thread may be finished nicely at the ends. (For method of calculating the depth of a thread see Chapter X.)

L. H. U. S. Std. means that a *left-hand* U. S. Standard Thread is to be cut. The diameter of this part of the work is $\frac{7}{8}$ in. From a table of threads the standard thread for $\frac{7}{8}$ in. diameter is found to be 9.

The bevel at the end of the work is put on after the thread is cut in order to finish the end of the thread properly.

It can easily be cut with the side of the thread tool after the thread is finished.

30. Tabulated Drawing—Taper Reamers. When several

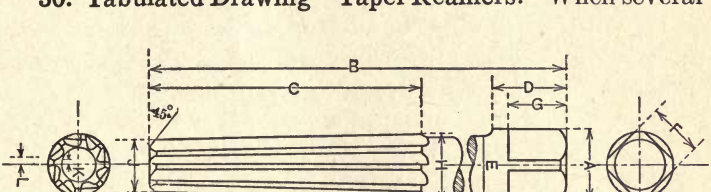


FIG. 30.—Morse Taper Reamer.

No.	1	2	3	4
A	.61	.80	1.03	1.22
B	$5\frac{3}{16}$	$6\frac{1}{8}$	$6\frac{7}{8}$	$8\frac{1}{8}$
C	3	$3\frac{1}{2}$	$4\frac{1}{4}$	$5\frac{1}{4}$
D	$\frac{3}{4}$	1	$1\frac{3}{16}$	$1\frac{5}{16}$
E	$\frac{7}{16}$	$\frac{5}{8}$	$\frac{7}{8}$	$1\frac{1}{8}$
F	.47	.62	.80	.95
G	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1
H	.519	.747	.991	1.293
J	.369	.572	.778	1.020
K	$\frac{3}{64}$	$\frac{1}{16}$	$\frac{5}{64}$	$\frac{5}{64}$
L	$\frac{1}{32}$	$\frac{3}{64}$	$\frac{1}{16}$	$\frac{1}{16}$
No. of Teeth	6	8	8	10
Taper per Foot	.600	.602	.602	.623
Taper per Inch	.050	.05016	.05016	.05191

pieces of work are of the same general pattern and differ only in length, diameter, etc., a tabulated drawing is sometimes made which will do for all the pieces of work.

Dimensions on the tabulated drawing, Fig. 30, are given

in *letters*, and the actual dimensions are given in tabulated form below the drawing. In the sketch we may note the following:

Numbers 1, 2, 3, 4, at the top of the table refer to the different sized reamers to be made. Let us suppose that we are to make a No. 3 Morse taper reamer.

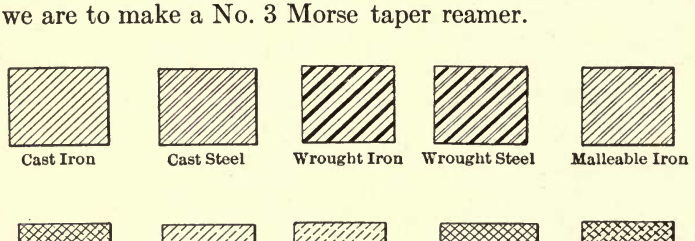


FIG. 31.—Materials shown in Cross-Section.

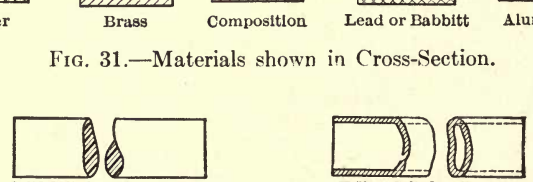


FIG. 32.—Forms for Representing Breaks.

Dimension **A** is the size of the square head from corner to corner. On the line **A** in the table under column No. 3, we find that for this reamer the length of the dimension **A** is 1.03 in. In the same manner dimension **B**, which is the total length of the reamer, is given under column No. 3 as $6\frac{7}{8}$ ins.; dimension **C**, which is the length of the

tapered part, is given as $4\frac{1}{4}$ ins., and so on. We may thus find from column No. 3 the length of every part of a No. 3 reamer.

For a No. 1 reamer column No. 1 should be read for every letter.

Reamers are made a little larger in diameter than the sizes given in order to leave material for grinding after the reamer is hardened. Thus the two diameters **H** and **J** are each left .015 in. too large.

31. Some Conventions Used on Blueprints. Figs. 31 and 32 give some conventional ways of showing materials and representing breaks. Fig. 33 means that the curve to be made has a radius as indicated, that is, the curve is a part of a circle with that radius. The center of the circle is indicated by a dot or by a straight line drawn as a radius.

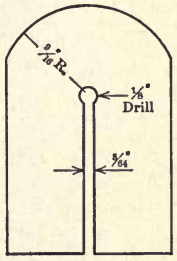


FIG. 33.

10 P. placed on a thread means that 10 threads per inch are to be cut. Other numbers of threads are indicated in the same way.

14 P. placed on a *gear blank* refers to the pitch system on which the tooth is cut, that is, it refers to the *diametral pitch* and indicates the size of tooth.

A *single circle* ◯ indicates a drilled hole, *two circles* ◎ indicate a countersunk hole.

QUESTIONS

1. How many center lines would be shown on an eccentric piece of work?

2. In the sketch for the bush plate, Fig. 26, does it mean any more to write a dimension as .750 in. than it does to write it .75 in.?

3. Would the same rules apply to the calculation of an inside taper as apply to an outside taper?

4. Mention some of the advantages of a tabulated drawing.

5. What is the meaning of: 18 P., R. H. U. S. Std. Thd.?

6. Is there any difference in the meaning of the sketches shown in Fig. 34?

7. In making measurements which may have been omitted on a blueprint, from what line should they always be taken?

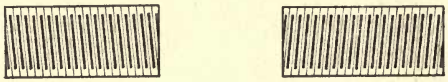


FIG. 34.

8. Is it better to measure missing dimensions on a blueprint with a scale, or to calculate them from given dimensions?

9. What is meant by "Scale Full Size" on a blueprint?

10. What is meant by "Scale: $\frac{3}{4}$ in. = 1 ft.?"

11. What is meant by a "perspective" drawing?

12. What is a "projection" in drawing?

13. When no center is given for the radius of a curve, but a straight line is drawn, where should the center be taken?

14. A "center line" is usually shown near the outside of a gear blank. Of what is this line the center?

CHAPTER V

MEASUREMENTS

The Straight Line and Circle

32. Use of the Machinist's Rule. The machinist's rule, Fig. 35, is divided on one side into 8ths and 16ths and on the other side into 32ds and 64ths of inches. Some rules have 100ths instead of one of the other markings. It is very necessary, in using the rule, to get it in a proper position to read any given length accurately. A good plan is to get the end of the work and the end of the rule against some flat surface as shown in the sketch. This will bring the zero mark of the rule exactly opposite the end of the piece of work to be measured. To read the mark on the rule at the other end of the piece of work, the eye must be placed exactly opposite the mark. If the eye is placed above or below the mark, the

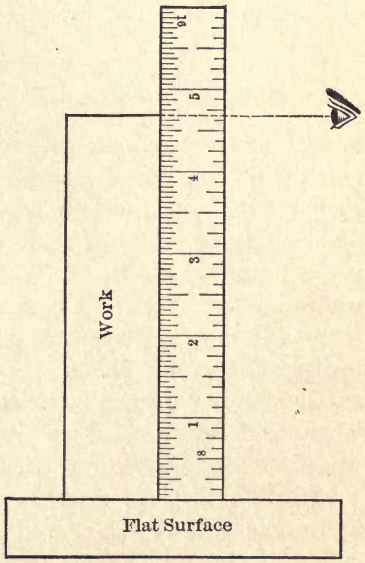


Fig. 35.

piece of work will seem shorter or longer than its true length.

The line on the rule has some thickness and we must always measure to the middle of this line.

33. Use of the Try Square. Suppose we are using a try square to test the corner of a metal block. Care must be taken to first bring the *beam* of the square against the side of the block **A**. This side must be perfectly flat and straight. The square is then pulled gently downward until the *blade* just touches the top of the metal block at **B**. The eye must be placed on a level with the top **B**, and it is best to look toward a strong light, when any uneven spots will show under the blade. Never pull the blade of the try square down hard against the top of the block as this will tend to spring the square out of shape. A common error made by machinists is to tip the square sidewise when testing; this should never be done, as the blade may not be straight when held sidewise. The proper way to hold the blade is vertically.

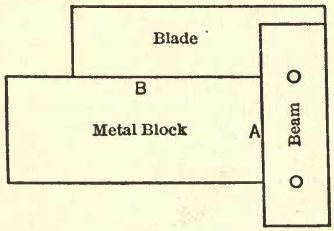


FIG. 36.—Use of a Try Square.

34. The Circle. A **circle** is a figure bounded by a curved line called the **circumference,** every point of which is equally distant from a point within called the **center.**

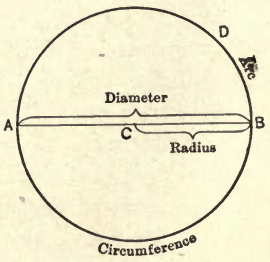


FIG. 37.

An **arc** is any portion of the circumference, such as **DB.**

The **radius** of the circle is any line drawn from the center to the circumference, as **CB**.

A **diameter** is a line drawn through the center of the circle and limited by the circumference, as **AB**. It is equal to *twice* the radius.

Every circle is divided into 360 equal parts called **degrees.**

In a series of circles with a common center there is always the *same number* of degrees between any two radii, such as **CA** and **CB**, Fig. 38, although the degrees of the outside circles are larger.

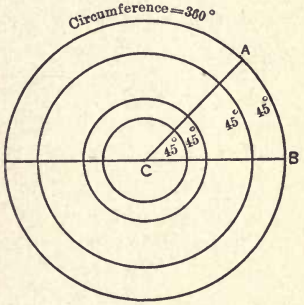


FIG. 38.

35. To Find the Circumference of a Circle. The circumference of a circle is always equal to about $3\frac{1}{7}$ (more accurately 3.1416) times the diameter. This may be

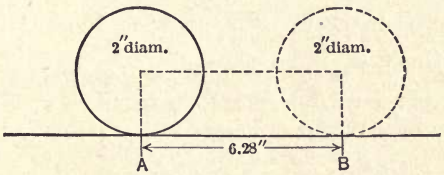


FIG. 39.

proved roughly in the following way: Take a piece of round stock about 2 ins. in diameter and place it on a flat surface. Mark the stock and the surface where the two touch as at **A**, Fig. 39. Now roll the stock through one complete turn until the mark on the stock is again at the

bottom as at **B**. Mark the surface at this point. The circumference has passed over the distance **AB**. Carefully measure this distance and divide it by the diameter of the stock. The result will be the relation of the circumference to the diameter of a circle. In the figure given, Fig. 39, $6.28 \div 2 = 3.14$.

When the diameter of a circle is given we may find its circumference by simply multiplying the diameter by 3.1416 (for most purposes $3\frac{1}{7}$ is accurate enough.)

Example. Find the circumference of a gear blank 3 ins. in diameter.

Solution. $3 \times 3.1416 = 9.4248$ ins. Ans.

PROBLEMS

1. What is the circumference of a round bar of iron 2 ins. in diameter?

2. The inside diameter of an engine cylinder is 5 ins. What is the circumference of the inside of the cylinder?

3. A piece of work 14 ins. in diameter is being turned in a lathe. What distance does the lathe tool cut in one revolution of the work?

4. A French engine cylinder was found to be 11.43 centimeters in diameter.

(*a*) What is the circumference in centimeters?

(*b*) If there are 2.54 centimeters in an inch, what would be the diameter of the cylinder in inches?

(*c*) What would be the circumference of the cylinder in inches?

5. A piece of wrought-iron pipe has an outside diameter of 1.9 in. and an inside diameter of 1.611 in. Find the outside and inside circumference of the pipe.

6. The circumference of a piece of pipe outside is 2.6389 in. What is the outside diameter?

7. Find the distance around a pulley 18 ins. in diameter.

36. The Use of Calipers. A convenient way of measuring the diameter of a cylindrical piece of work is by the use

of calipers. The calipers are held as shown in Fig. 40. In order to make an accurate measurement, the calipers must be so finely adjusted that they will just touch the sides of the work as they are passed over it. They should be held at the top and should be set with just enough pressure on the sides of the work so that they will slip over by their own weight. The distance between the legs of the calipers may be found by placing one leg at the end of a machinist's rule and reading the mark on the rule at the other leg.

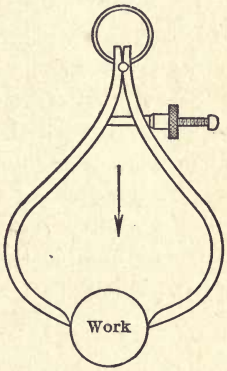


FIG. 40.

Calipers are also used to test whether a piece of work is parallel or not. This is done by trying at different positions along the piece of work as in Fig. 41. When

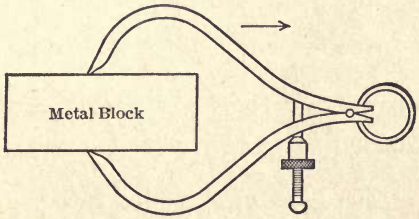


FIG. 41.

testing in this way, care must be taken not to turn the calipers sideways. The best method is to hold them in the hand so that the top touches the palm of the hand and the two legs rest lightly on the fingers. The calipers

are then resting on three points which will allow them to turn to the right position. The points must always be kept in good condition and be of the right shape. The joint at the top must also be firm.

37. The Micrometer Caliper. The micrometer caliper is an instrument which is used for very accurate measurements. It consists of a solid frame, Fig. 42, at one side of which is a fixed nut. In the fixed nut is a freely moving screw which has 40 threads to the inch. The *hub* is graduated 40 to the inch so that *one complete turn* of the screw makes the *sleeve* travel $\frac{1}{40}$ in. or .025 in. along the hub, showing

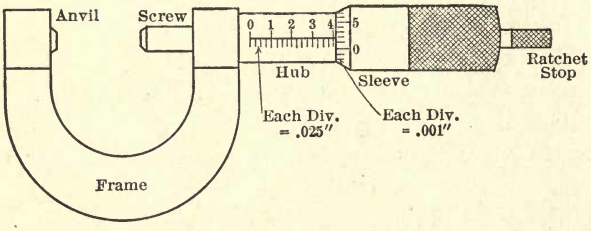


FIG. 42.—The Micrometer Caliper.

that the caliper has opened that amount. Every *fourth* graduation on the hub is numbered 1, 2, 3, 4, etc., so that each of these marks is $4 \times .025$ in. or .100 in. The beveled edge of the sleeve is graduated into 25 equal parts; each of these equal parts would therefore be $\frac{1}{25}$ of one turn or $\frac{1}{25} \times .025$ in. $= .001$ in.

To see if the caliper is properly adjusted it should be closed completely. This should be done slowly and with pressure enough to give one click of the *ratchet stop* at the end of the sleeve. If the 0 mark on the sleeve stops at the 0 mark on the hub and exactly on the line running

along the axis of the hub, the micrometer is properly adjusted. If it is not on the zero mark, the *anvil* at the other side of the frame may be adjusted so as to bring the sleeve to the proper position.

38. To Read the Micrometer Caliper.

CASE I

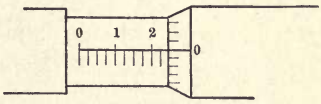


FIG. 43.

It is easiest to count the lines marked 1, 2, 3, etc., on the hub in thousandths, as 100, 200, 300, etc. The next lines on the hub are counted also in thousandths, as 25, 50, 75, etc. In this case we have therefore:

200+50=.250″. Ans.

Hub:

Large divisions	.200
Small divisions	.050
	.250″

Ans.

CASE II

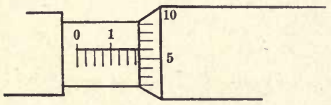


FIG. 44.

Hub:

Large divisions	.100
Small divisions	.075
Sleeve	.006
	.181″

Ans.

In this case we have on the hub 100+75=.175″, and on the sleeve we have 6 or .006″. The total reading is therefore:

100+75+6=.181″. Ans.

Case III

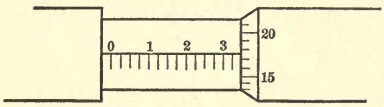


Fig. 45.

Hub:	
Large divisions	.300
Small divisions	.025
Sleeve	.017½
	.342½″
Generally written	.3425″

On the hub the reading is 300+25=.325″, and on the sleeve the reading is 17 and a little over. The exact mark is half way between 17 and 18 or 17½ thousandths. The total is

$$325+17\tfrac{1}{2}=.342\tfrac{1}{2}''.$$

Ans.

When using the micrometer caliper, it should always be opened a little larger than the piece to be measured, and then closed *slowly* until it touches the piece of work with *one click* of the ratchet stop.

QUESTIONS

1. What fractional part of an inch is a micrometer caliper opened by making one complete turn of the sleeve? What decimal part of an inch?

2. If the sleeve is turned through 10 complete revolutions, what part of an inch is the micrometer opened?

3. Through how many turns must the sleeve be moved to open the micrometer .5 of an inch?

4. What part of an inch is indicated by the first line beyond 0 on the hub?

5. What part of an inch is indicated by the line numbered 3 on the hub?

6. The hub reading is 3 and the sleeve reading is 12. What is the size of opening of the micrometer?

7. The hub reading is 8 and the sleeve reading is 12½. What is the size of opening of the micrometer?

8. How many turns must the micrometer be opened to read .375 in.?

9. How would you set a micrometer to read .4375 in.?

10. Reduce $\frac{9}{16}$ in. to a decimal and tell how you would set a micrometer to read this amount?

39. The Vernier Caliper. The vernier caliper, Fig. 46, consists of a *bar* with a *sliding jaw*. The bar is divided into *inches* and *tenths* of inches, while each *tenth* of an inch

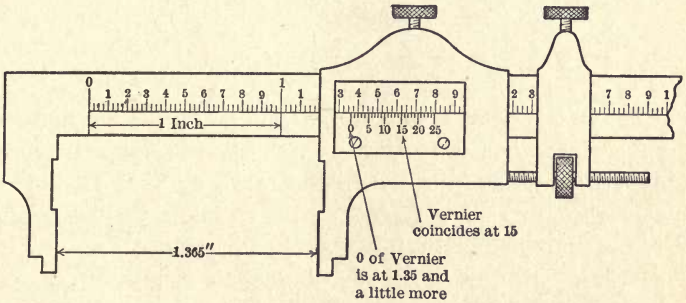


FIG. 46.—The Vernier Caliper.

is divided into *four* parts. The smallest divisions on the bar are therefore each $\frac{1}{40}$ of an inch or .025 in.

On the sliding jaw is a line of division called a **vernier** (from the name of the man who invented it). This vernier is divided into 25 parts numbered 0, 5, 10, 15, 20, 25 and the *whole length* of these *25 parts* is equal to *24 divisions* (40ths of an inch) on the bar.

Since the *25 parts* of the vernier cover but *24 parts* (each $\frac{1}{40}$ in.) on the bar, *one* of these 25 parts would therefore be $\frac{1}{25}$ of $\frac{1}{40}$ or $\frac{1}{1000}$ in. *shorter* than each division on the bar. If the 0 of the vernier is placed opposite the 0 on the bar, the *first line* on the *vernier* will then be $\frac{1}{1000}$ in. behind the *first line* on the *bar*, the 5th line on the vernier will

be $\frac{5}{1000}$ in. behind the 5th line on the bar, and so on, until we get to line 25 on the vernier, which will be opposite line 24 on the bar. If we now advance the vernier $\frac{1}{1000}$ in., the line 1 on the vernier will be opposite line 1 on the bar; if the vernier is advanced $\frac{5}{1000}$ in., line 5 of the vernier

FIG. 47.—The Vernier.

will be opposite line 5 on the bar and so on. Each line on the vernier when opposite a line on the bar therefore adds as many thousandths as the mark on that particular line of the vernier. In reading the vernier we look along until we find a line that is exactly opposite a line on the bar (there will be only one line that exactly matches) and then we read the mark on the vernier, *not* on the bar.

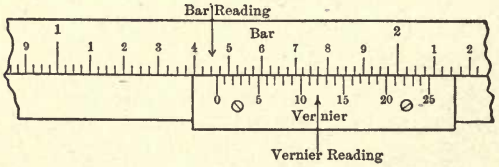


FIG. 48.

Bar:	
Whole inches	1.
Tenths	.4
Thousandths	.050
Vernier	.012
	1.462″. Ans.

The easiest way to read the vernier caliper is as follows: First read the whole inches on the bar, in this case 1; then 10ths of inches, in this case .4; then thousandths of inches to the 0 of the vernier, in this case .050″. To this we add the reading of the vernier where it exactly matches a line on the bar, in this case 12 or .012″. The total reading is therefore

$$1+.4+.050+.012=1.462''. \quad \text{Ans.}$$

QUESTIONS

1. How would you set a vernier caliper to read $\frac{5}{8}$ in.?
2. What part of an inch is indicated by the 10th line from 0 on the bar of the vernier caliper?
3. How would you set a vernier caliper to read .5625 in.?
4. How would you set a vernier caliper to read $\frac{11}{16}$ in.?
5. What part of an inch is represented when the bar reads 3 lines beyond the inch and the vernier reading is 5?
6. Are the readings with a vernier caliper as easy as with a micrometer? Are they as accurate?
7. Reduce $\frac{7}{8}$ in. to a decimal and set the vernier caliper to correspond with this measurement.

CHAPTER VI

MEASUREMENTS

Surfaces

A **rectangle** is a plane surface bounded by four straight lines and having four right angles.

40. To Find the Area of a Rectangle. Example. The top of a bench is 5 ft. long and 3 ft. wide. Find its area.

If the length of the bench is measured off into 5 separate

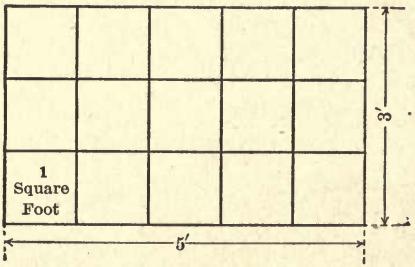


Fig. 49.

feet, and the width into 3 separate feet, and lines are drawn through the distances marked off, the rectangle will be divided into small squares each containing *one* square foot, Fig. 49. The whole rectangle contains 15 or 5×3 of these squares. This represents the number of square feet in the rectangle. The *area* of a rectangle

may therefore be found *by multiplying its length by its width.*

Area of a rectangle = length × width.

PROBLEMS

1. What is the area of the floor of a shop 40 ft. long and 20 ft. wide?

2. Find the number of square inches in a piece of sheet iron 36 ins. long and 18 ins. wide.

3. A block of cast iron on a planer is 3 ft. long and $2\frac{1}{2}$ ft. wide. How much surface is there to be planed on the top of the block?

4. The top of a bench contains 20 sq.ft. If the bench is $2\frac{1}{2}$ ft. wide, how long is it?

5. A boy chips the top of a block of cast iron which is 4 ins. long and $2\frac{1}{2}$ ins. wide. How many square inches of metal does he chip?

A **triangle** is a plane figure bounded by three straight lines and having three angles. If one of these angles is a *right angle*, the triangle is a *right-angled triangle.*

41. To Find the Area of a Right-angled Triangle. If a line be drawn from one corner to the opposite corner of a rectangle, the rectangle will be divided into two equal right-angled triangles, Fig. 50. The *area* of one of these triangles is evidently *one-half* of the area of the rectangle. The **base** of the right-angled triangle is the same as the *length* of the rectangle, and the **altitude** of the right-angled triangle is the same as the width of the rectangle. The *area* of the right-angled triangle is therefore:

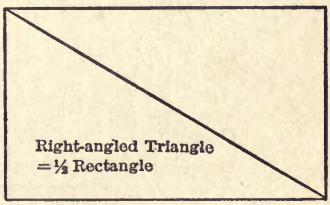


FIG. 50.

Area of right-angled triangle = $\frac{1}{2}$ × base × altitude.

It may be proved by geometry that this statement is true for any triangle. The **altitude** of any triangle is the *perpendicular distance* from the base to the *top* or **vertex** of the triangle.

PROBLEMS

1. How many square inches in a right-angled triangle, whose base is 8 ins. and whose altitude is 6 ins.?

2. Find the area of a right-angled triangle whose base is 4 ins. and whose altitude is 3 ins.

3. Find the area of an equal-sided triangle whose base is 7 ins. and whose altitude is 6 ins.

4. A triangle contains 270 sq.ins. If the base is 36 ins., what is the altitude?

5. A triangle contains 150 sq.ins. If the altitude is 12 ins., what is the length of the base?

42. To Find the Area of Figures of Different Shapes. Figures may often be divided into rectangles and triangles.

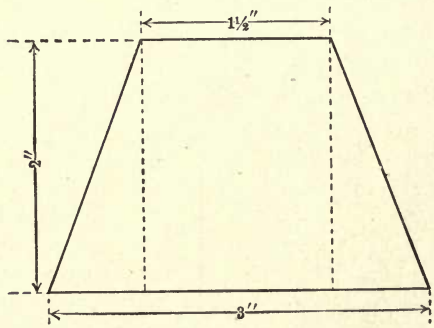


Fig. 51.

The sum of the areas of the separate parts will be the total area of the figure. The method of division is shown in Fig. 51.

PROBLEMS

1. Find the total area of Fig. 52.

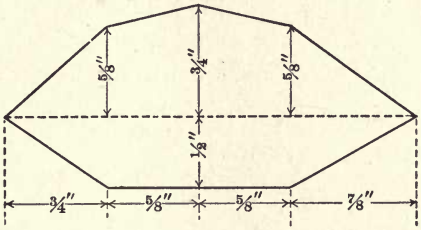


FIG. 52.

2. Find the area of Fig. 53.

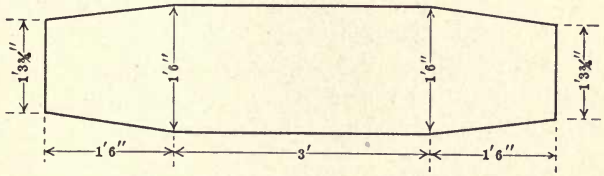


FIG. 53.

3. What is the area of figure 54?

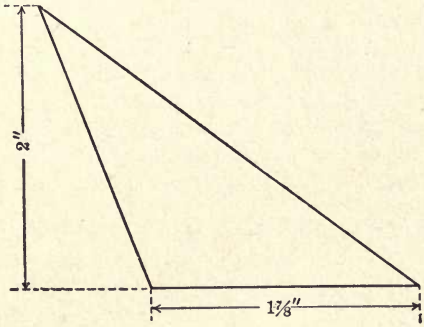


FIG. 54.

43. To Find the Area of a Hexagon. If lines are drawn from the center of the hexagon **O**, to the points, **A, B, C, D, E, F**, Fig. 55, the figure will be divided into *six* equal triangles. The *area* of each triangle is equal to $\frac{1}{2} \times$ base $\times$ altitude and the sum of these six areas will be the area of the hexagon. If we should multiply by *six* bases instead of *one*, the result would be *six times* as large as the area of one triangle; but *six bases* would be the length around the whole figure. This length is called the **perimeter**. The *altitude* of one triangle **OX** is called the **apothem** of the hexagon. If we take six bases as the *perimeter* and the altitude as the *apothem* we may find the area of the hexagon:

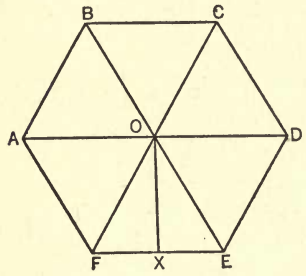


FIG. 55.

$$\text{Area of hexagon} = \tfrac{1}{2} \times \text{perimeter} \times \text{apothem}.$$

PROBLEMS

1. Find the area of a hexagon whose perimeter is 6 ins. and whose apothem is .866 in.

2. The side of a hexagon nut measures $\frac{1}{2}$ in., while the distance from the center of the nut to the center of the side is .433 in. Find the area of the top face of the nut.

3. The length of one side at the end of a hexagonal steel bar is $\frac{3}{4}$ in. If the apothem of the bar is .6495 in., what is the area of the end of the bar?

4. The side of a hexagon nut measures 1.5 in. If the apothem is 1.299 in., what is the area of the top face of the nut?

44. To Find the Area of a Circle. A circle may be considered as being made up of a large number of triangles

as shown in Fig. 56. If the number of these triangles is increased, the *bases* of the triangles will gradually become smaller and fit more closely to the *circumference* of the circle. We may imagine the bases to become so small that they *practically become the circumference of the circle.* The *altitude* of any one triangle will then be the *radius* of the circle. We have just seen that the *area* of a hexagon is

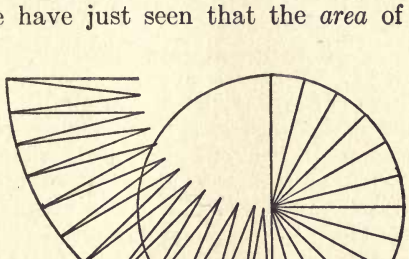


FIG. 56.

equal to $\frac{1}{2}\times$perimeter$\times$apothem. In the circle the *circumference* is the *perimeter* and the *apothem* is the *radius.* We may then say that:

$$\text{Area of circle} = \tfrac{1}{2}\times\text{circumference}\times\text{radius.}$$

But the circumference of a circle = diameter $\times$ 3.1416, and the radius = $\frac{1}{2}$ diameter. We may now write the above statement:

$$\text{Area of circle} = \tfrac{1}{2}\times(\text{diam.}\times 3.1416)\times(\tfrac{1}{2}\text{ diam.})$$

Or, by changing the order of the expression,

$$\text{Area of circle} = \tfrac{1}{2}\times\tfrac{1}{2}\times\text{diam.}\times\text{diam.}\times 3.1416,$$

that is:

$$\text{Area of circle} = \frac{(\text{diam.})^2 \times 3.1416}{4},$$

or

$$\text{Area of circle} = (\text{diam.})^2 \times .7854.$$

Example. Find the area of a circle 4 ins. in diameter.

Solution. $(4)^2 \times .7854 = 16 \times .7854 = 12.5664$ sq.ins. Ans.

PROBLEMS

1. Find the area of a circle 6 ins. in diameter.
2. What is the area of a circle $4\frac{1}{2}$ ins. in diameter?
3. Find the area of the end of a steel bar $1\frac{1}{2}$ in. in diameter.
4. What is the area of cross-section of the inside of a steam pipe $2\frac{1}{2}$ in. in diameter?
5. Which has the larger area of cross-section, a square bar of steel measuring 2 ins. on each side, or a round bar $1\frac{3}{8}$ in. in diameter?
6. A square box trough is to be built to feed water into a drain pipe which is round and 8 ins. in diameter. What is the nearest size of square that will have the same area of cross-section as the pipe?
7. A rectangular leader measuring 2×4 in. inside, carries water into a round drain pipe 3 ins. in diameter. Which will carry more water, the leader or the drain pipe?

45. Angles. An **angle** is the opening between two straight lines drawn from the same point.

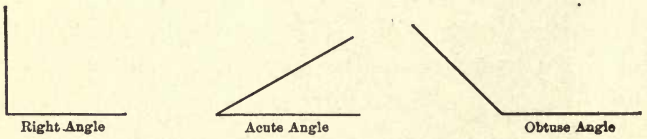


FIG. 57.

A **right angle** is made when one straight line is perpendicular to another straight line.

An **acute angle** is less than a right angle.

An **obtuse angle** is greater than a right angle.

46. The Draughtsman's Triangle. The draughtsman's triangle is used for laying out angles which frequently occur. It is made of steel, hard rubber, or celluloid. Two convenient shapes are shown in Fig. 58; one having angles of 90 degrees, 45 degrees, and 45 degrees, while the other has angles of 90 degrees, 60 degrees, and 30 degrees.

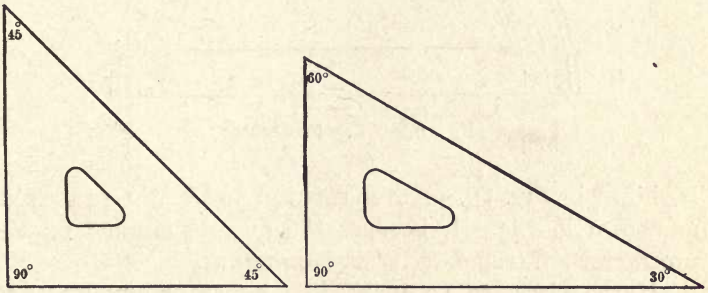


FIG. 58.—Draughtsman's Triangle.

By combining the two triangles in different positions angles of 75, 105, 120, 135, and 150 degrees may be obtained.

47. The Protractor. For marking out angles other than those given on a draughtsman's triangle the protractor is used. A simple form of this instrument is shown in Fig. 59. The *bottom* of the protractor is the diameter of a circle whose degrees are marked out on the *top* of the protractor. The whole protractor would cover one-half of the circle. A *notch* cut in the bottom of the protractor marks the *center* of the circle.

In using the protractor care must be taken to have the bottom or *base line* exactly on the line from which

the angle is to be marked, with the center notch exactly at the *point* in that line from which the line marking out the angle starts. The method of marking an angle

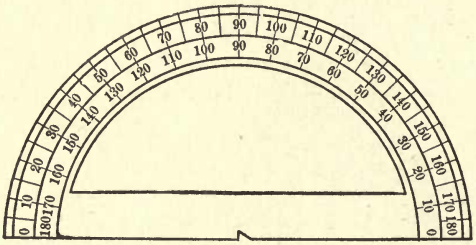


FIG. 59.—The Protractor.

is shown in Fig. 60. Never attempt to use the protractor as shown in Fig. 61, because the degrees marked on the protractor are true only for a given radius.

If the circle to be marked is larger or smaller than

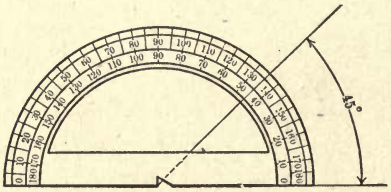


FIG. 60.

the protractor, the protractor must always be placed at the *center of the circle*. The degrees may then be marked on any size circle by extending lines across the protractor.

48. The Bevel Protractor. A convenient form of protractor for the machinist is the bevel protractor shown in

Fig. 62. It consists of a base with a *dial* attached which is graduated in degrees. This dial can be turned at any

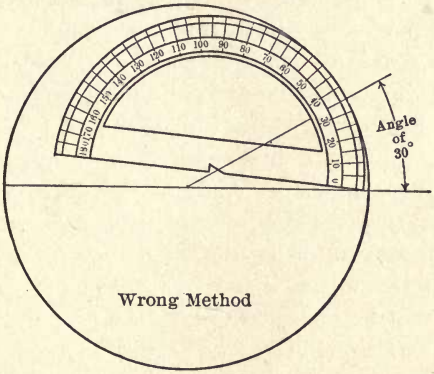


Fig. 61.

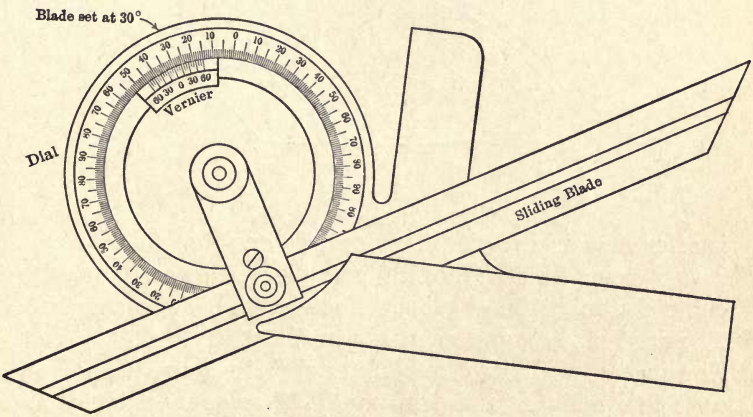


Fig. 62.—Bevel Protractor.

angle and has a thumb nut at the center which clamps it in any position. Just below the dial is a *sliding blade*

which may be adjusted to different lengths independently of the dial. The back of the bevel protractor is flat and allows it to be placed directly on the surface to be marked. Since the dial reads in both directions and the blade can be adjusted to almost any length, it will readily be seen that this protractor can be used in almost any position.

In using the bevel protractor care must be taken to see that the base line and blade make the angle desired and not some other angle. A common mistake is to get the *complementary angle* which is explained on page 67. The angle desired may often be obtained by working backward from the 90-degree mark, and it is sometimes necessary to turn the protractor over to mark the proper angle.

49 (*a*). The Bevel Protractor Vernier. A vernier, Fig. 63,

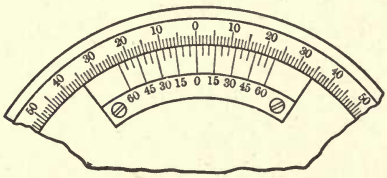


FIG. 63.

is often attached to the bevel protractor. For fine work in angles, degrees are divided into minutes and seconds. Every circle contains 360 degrees. Each degree is divided into 60 minutes (60′) and each minute into 60 seconds (60″). The *vernier* is divided into minutes with a line drawn at each five minutes and with every 15 minutes marked, as 15, 30, 45, 60. The reading of the vernier may be taken either to the right or to the left of the 0 mark on the vernier, according to which way the protractor is moved.

Every space on the vernier is 5 minutes shorter than

two spaces on the dial scale. When the 0 of the vernier is opposite the 0 on the dial scale, the edges of the blade and the base of the protractor are parallel. When line 1 (5′ line) on the vernier is exactly opposite line 2 (2° line) on the dial scale, the *angle* between the base and the blade has been changed 5′; when line 6 (30′ line) on the vernier is exactly opposite line 12 (12° line) on the dial scale, the angle between the base and the blade has been changed 30′ and so on.

49 (*b*). To Read the Setting of the Protractor with the Vernier. First read the whole degrees on the dial scale

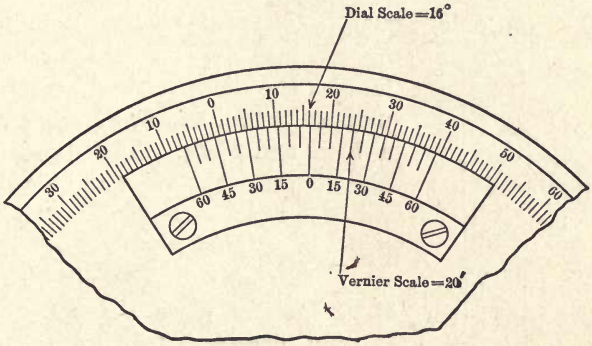


FIG. 64.

between the 0 mark on that scale and the 0 mark on the vernier. In Fig. 64 it is 16°. Next read in the same direction on the *vernier* to a line which is exactly opposite a line on the dial scale. Since the vernier counts by 5's the line in this case is 20 minutes. The whole reading is therefore 16° 20′.

QUESTIONS

1. The blade and base of a bevel protractor are parallel when the dial scale reads 0. If the dial is turned through a right angle, what will be the reading of the dial scale?

2. What will be the reading of the dial scale if the blade and base are set at $\frac{1}{2}$ a right angle?

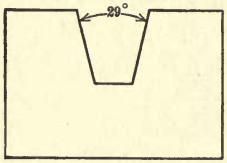


FIG. 65.

3. How would you set the bevel protractor for an angle of 150°?

4. How would you set the bevel protractor for an angle of 120°?

5. A templet is to be marked out for a 29° thread as in the sketch, Fig. 65. At what angle should the bevel protractor be set for each side of the tooth?

50. The Center Square. The center square is an instrument which can be used for quickly finding the diameter and center of a piece of round stock. It consists of a *head* and *blade*. The head is so placed that the edge of the blade will come across the diameter of a piece of round stock placed in the head, as shown in Fig. 66. To find the *center* of a piece of round stock it is only necessary to turn the stock to some other position and draw

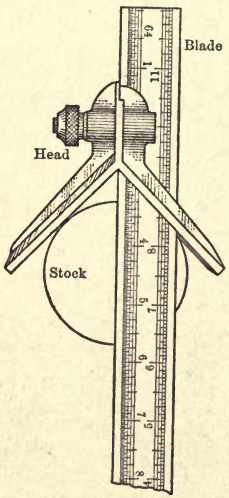


FIG. 66.

FIG. 67.

another diameter. Where the two diameters cross as in Fig. 67, will be the center of the piece of stock.

REVIEW PROBLEMS

1. Find the area of a circle $\frac{1}{2}$ in. in diameter.

2. Which will have the greater area, a square measuring 8 ins. on a side, or a circle 9 ins. in diameter?

3. What is the area of cross-section of an iron pipe measuring $1\frac{1}{4}$ ins. inside diameter?

4. What is the area of a triangle whose base is 10 ins. and whose altitude is 8 ins.?

5. How many times larger is a square which measures 9 ins. on a side than one which measures 6 ins. on a side?

6. How many square feet of floor space will be occupied by a milling machine which takes 108 ins. at right angles to the spindle, and 90 ins. parallel to the spindle?

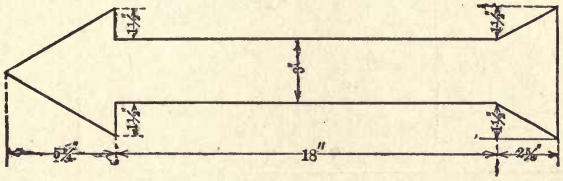


FIG. 68.

7. The side of a hexagon is $1\frac{1}{4}$ in. and the apothem is 1.1025. Find the area of the hexagon.

8. If the perimeter of a rectangle is 72 ft., and the length is equal to twice the width, find the area.

9. A gas engine has a piston 5 ins. in diameter. When the pressure in the cylinder is 200 lbs. per square inch, what is the total pressure on the piston?

10. Find the area of Fig. 68.

11. Draw a straight line 6 ins. long. With a protractor lay off at the center of the line an angle of 130°. How many degrees are there in the remaining angle on the top of the line?

12. Compare the area of a hexagon with a side 1 in. long and an apothem of .866 in. with the area of a circle $1\frac{5}{8}$ in. in diameter.

CHAPTER VII

CONSTRUCTIONS

51. To Find the Center of a Straight Line. Suppose that the line **AB**, Fig. 69, is the line of which you are to find the center. Set a pair of dividers to a distance which is a little more than one-half the line **AB**. Take **A** as a center and draw arcs at **C** and **D**. With the same

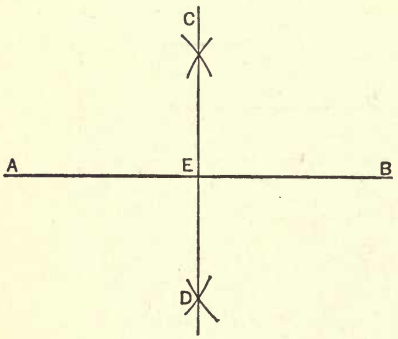


FIG. 69.

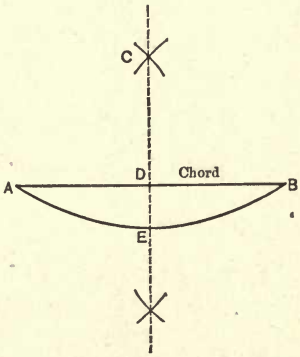


FIG. 70.

setting of the dividers, take **B** as a center and draw arcs at **C** and **D**, cutting the arcs already drawn. Connect the point of intersection of the arcs at **C** with the corresponding intersection at **D**. Where the line **CD** crosses **AB** at **E** will be the center of the line **AB**.

52. To Find the Center of an Arc of a Circle. Connect the ends of the arc, Fig. 70, by a straight line **AB** (called a

chord). Find the middle of the line **AB** by the same method as in Section 51. The line **CD** will also divide the arc **AB** into two equal parts. The point **E** is therefore the center of the arc **AB**.

PROBLEMS

1. Draw a line about 6 or 7 ins. long and find its center by the method of Section 51.
2. Draw a circle of 3 ins. radius, mark off a chord about 3 ins. long, and find the center of the arc thus made.
3. Get a piece of stock which has been finished in the shaper and find the center of its top surface, in the direction of its length, by the method of Section 51.

53. To Draw a Perpendicular from a Point to a Line.

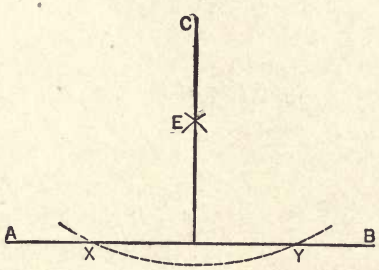


FIG. 71.

Suppose that **C** is the point, and **AB** is the line, Fig. 71. From **C** as a center draw an arc cutting the line **AB** at **X** and **Y**. From **X** and **Y** as centers draw arcs cutting each other at **E**. Draw a line from **C** through **E** to the line **AB**. This line will be the perpendicular required.

54. To Erect a Perpendicular at a Point in a Straight

Line. Case 1. *When the point is in the line as at* **C**, Fig. 72.

With the point **C** as a center, draw an arc cutting the line **AB** at **X** and **Y**. With **X** and **Y** as centers draw arcs cutting each other at **O**. A line drawn from **O** to **C** will be the perpendicular required.

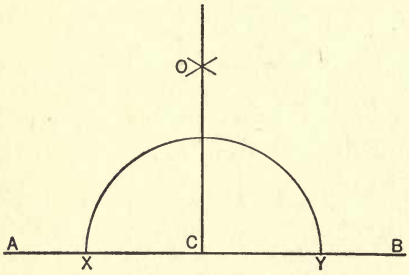


Fig. 72.

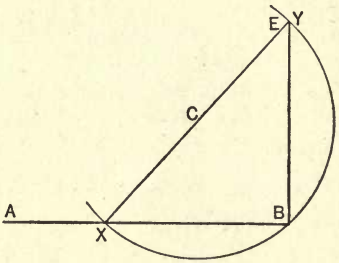


Fig. 73.

Case 2. *When the point is at the end of the line,* Fig. 73.

Given the line **AB**. It is required to draw a perpendicular at the point **B**.

Take any point outside the line such as the point **C**. With **C** as a center and a radius equal to **CB**, draw the arc **XY**. Draw **XC** and continue it to **E**. A line drawn from **E** to **B** will be the perpendicular required.

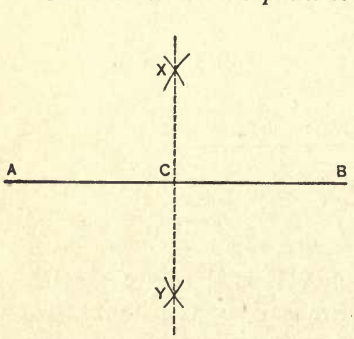


Fig. 74.

Case 3. *To draw a perpendicular at the middle of a straight line,* Fig. 74.

It is required to draw a perpendicular at the middle

of the straight line **AB**. With **A** and **B** as centers draw arcs cutting each other at **X** and **Y**. Draw the line **XY**, which will be perpendicular to the line **AB** at its middle point **C**.

55. Angles. If two straight lines **AB** and **XY** are perpendicular to each other as in Fig. 75, there will be *four* 90° angles or *right angles* around the point **O** where the lines cross. The greatest angular measure around any point is four right angles or 360°.

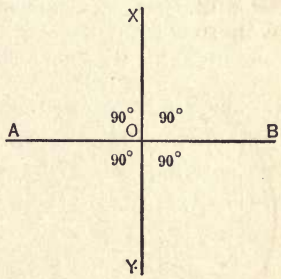


Fig. 75.

If we consider one side of the straight line **AB**, we see that the greatest angular measure on *one side* of a straight line is *two* right angles, or 180°.

When the sum of *two* angles is equal to 90° as in Fig. 76, the angles are said to be **complementary.** Each angle is the *complement* of the other angle, that is, 60° is the complement of 30° and 30° is the complement of 60°.

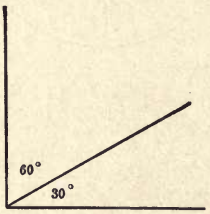


Fig. 76.

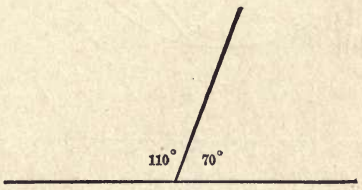


Fig. 77.

When the sum of *two* angles is equal to 180° as in Fig. 77, the angles are said to be **supplementary.** Each angle

is a *supplement* of the other angle, that is, 70° is the supplement of 110° and 110° is the supplement of 70°.

QUESTIONS

1. What is the complement of an angle of 40°?
2. What is the supplement of an angle of 50°?
3. What is the complement of an angle of 50°?
4. How many degrees are there in three right angles?
5. What is the supplement of an angle of 90°?

56. To Find the Center of a Circle. From any point in the circumference such as **A**, draw any two *chords* as **AB** and **AC**, Fig. 78. Draw perpendiculars at the middle of these chords by the method of Section 54, Case 3. Where

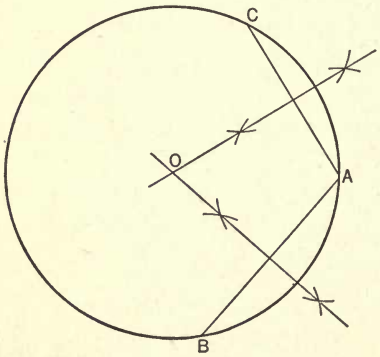


Fig. 78.

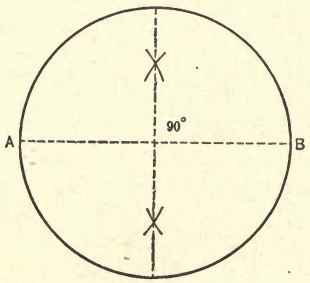


Fig. 79.

these two perpendiculars cross at **O** will be the center of the circle.

57. To Divide a Circle into a Given Number of Parts.

Case 1. *To divide the circle into four equal parts,* Fig. 79.

Draw the diameter of the circle **AB**. At the middle point of the diameter draw a perpendicular by the method

of Section 54, Case 3. Since each angle is a right angle or 90°, the circumference of the circle will be divided into *four* equal parts, as there are 360° in the whole circumference.

If each of the four arcs of the circle is now divided into *two* equal parts by the method of Section 52, the circle will be divided into *eight* equal parts, Fig. 80. By continuing this process of dividing the arcs the circle may be divided into 16, 32, 64, etc., parts.

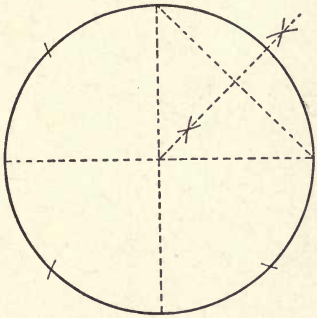


FIG. 80.

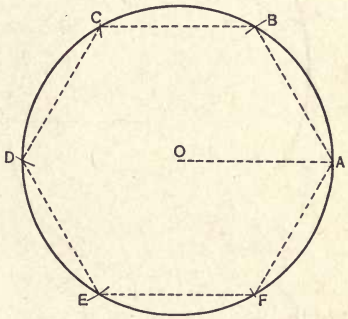


FIG. 81.

CASE 2. *To divide a circle into six equal parts,* Fig. 81.

Draw any radius as **OA**. With **A** as a center and a radius equal to **OA**, mark off a chord **AB** equal to **AO**. Next take **B** as a center and with the same radius mark off **BC**. Continue this process around the circle when equal arcs will be marked at points **A, B, C, D, E, F,** thus dividing the circle into six equal parts.

By dividing each arc into *two* equal parts by the method of Section 52, the circle will be divided into 12, 24, 48, etc., parts.

By taking every other point in Fig. 81, as **A, C,** and **E,** the circle will be divided into *three* equal parts.

CASE 3. *To divide a circle into ten equal parts*, Fig. 82.

Draw any radius as **OA**. Draw a horizontal line equal in length to the radius **OA**, as in Fig. 83. At the point **A** in this line draw a perpendicular **AC** and make it $\frac{1}{2}$ as long as **OA**. From **C** as a center, with a radius equal to **CA**, draw a circle. From **O** draw the line **OC** cutting the circle at **X** and **Y**. Then **OX** is the length of the chord

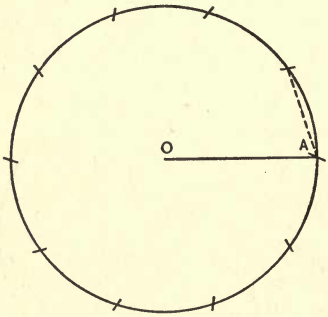


FIG. 82.

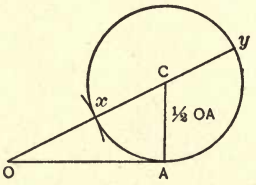


FIG. 83.

which will mark off $\frac{1}{10}$ of the circumference. With a pair of dividers set to this distance mark off ten equal chords and divide the circumference into *ten* equal parts.

If each arc is divided into *two* equal parts by the method of Section 52, the circle will be divided into 20, 40, etc., equal parts.

By taking every other point the circle will be divided into *five* equal parts.

REVIEW QUESTIONS

1. Draw a circle 3 ins. in diameter and divide it into 5 equal parts.
2. Draw a circle 2 ins. in diameter and divide it into 6 equal parts.

3. Draw a line 5 ins. long and draw a perpendicular at its middle point.

4. Draw a perpendicular at the end of a straight line 4 ins. long.

5. How many degrees are there in $\frac{3}{4}$ of a circle? How many right angles?

6. How would you draw an angle of 45°?

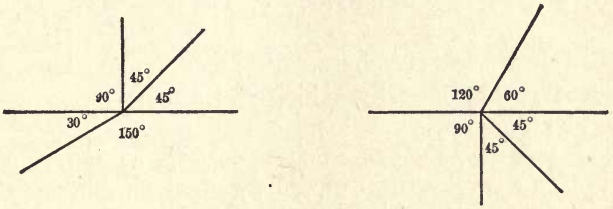


FIG. 84.

7. In the figures shown in Fig. 84 give as many complementary and as many supplementary angles as you can.

8. If lines are drawn from each of the corners of a hexagon to the center, how many angles are made at the center? How many degrees are there in each angle?

9. What is the complement of an angle of 45°? What is its supplement?

10. A hexagon is drawn outside a circle as in Fig. 85. The radius of the circle is the apothem of the hexagon. If the diameter of the circle is 3 ins., and the side of the hexagon is 1.7314 ins., compare the area of the circle with that of the hexagon.

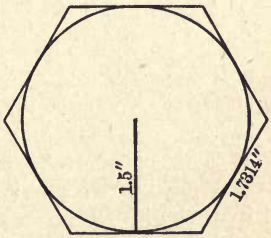


FIG. 85.

CHAPTER VIII

POWERS, SQUARE ROOT, SIGNIFICANT FIGURES, RIGHT-ANGLED TRIANGLE

58. Powers of Numbers. A **power** of a number is the product obtained by taking the number one or more times as a factor.

The **index** or **exponent** of a power is a figure placed at the right and a little above to show how many times it is used as a factor, thus:

$3 \times 3 = 9 = 3^2$, or the 2d power, or square of 3
$3 \times 3 \times 3 = 27 = 3^3$, or the 3d power, or cube of 3
$3 \times 3 \times 3 \times 3 = 81 = 3^4$, or the 4th power of 3
$3 \times 3 \times 3 \times 3 \times 3 = 243 = 3^5$, or the 5th power of 3

and so on.

PROBLEMS

1. What is the second power of 4?
2. What is the third power of 3?
3. How much is $(\frac{5}{8})^2$?
4. How much is $(\frac{1}{2})^5$?
5. What is the square of $2\frac{1}{4}$?
6. What is the second power of .2?
7. What is the third power of .5?
8. The second power of $(2a)$ is 2^2a^2 or $4a^2$. What is the second power of $2x$? Of $2y$?

59. Roots of Numbers. The **root** of a number is one of the equal factors which produce it.

A root is often indicated by a **radical sign** $\sqrt{\ }$.

The **index** of the root is a figure placed over the radical sign to show how many equal factors of the number are to be found.

$\sqrt{4}$ or $\sqrt[2]{4}$ denotes the square root of 4.

$\sqrt[3]{8}$ denotes the cube root of 8.

$\sqrt[5]{32}$ denotes the 5th root of 32.

PROBLEMS

1. What is the square root of 25?
2. What is the third root of 27?
3. What is the second root of 36?
4. What is the third root of 125?
5. What is the fourth root of 16?

60. To Find the Square Root of a Number. 1. *Division into periods.* The smallest integer of *one* order of figures is 1, and the largest 9; the smallest integer of *two* orders of figures is 10, and the largest 99, and so on.

$(1)^2 =$	1	$(9)^2 =$	81
$(10)^2 =$	100	$(99)^2 =$	9,801
$(100)^2 =$	10,000	$(999)^2 =$	998,001
$(1000)^2 =$	1,000,000	$(9999)^2 =$	99,980,001
etc.			

From the above table it will be seen that the square of any integer less than 10 has either *one* or *two* figures; the square of any integer less than 100 and over 9 has either *three* or *four* figures, etc. Therefore the *square* of any number consists of *twice* as many figures as the *root*, or one less than twice as many.

We may then find the number of figures in the square

root of a number by beginning at the right and marking the number off into periods of *two figures* each, thus:

The square root of 183,184 will have *three* figures, $\widehat{18}\ \widehat{31}\ \widehat{84}$.

The square root of 21,418,384 will have *four* figures, $\widehat{21}\ \widehat{41}\ \widehat{83}\ \widehat{84}$.

The square root of 9,703,225 will have *four* figures, $\widehat{9}\ \widehat{70}\ \widehat{32}\ \widehat{25}$.

2. **Example.** Find the square root of 133,225.

	$\widehat{13}\ \widehat{32}\ \widehat{25}$	(365 Ans.
	9	
Trial $3\times2=6$	) 432	
Complete $=66$	) 396	
Trial $36\times2=72$	) 3625	
Complete $=725$	) 3625	

1. Separate the number into periods of *two* figures each, beginning at the right.

2. Find the greatest square in the left-hand period (13), and place its root at the right of the number. In 13 the greatest perfect square is 9, and its square root is 3. Place 3 in the root.

3. Subtract the greatest square (9) from the first period (13) and bring down the next period with the remainder (4) making 432 for a new dividend.

4. Double the root already found for a trial divisor ($3\times2=6$), and divide it into the new dividend, omitting the right-hand figure, thus: $43\div6=6$, which is the next figure of the root. Place 6 in the root. Also write 6 after the trial divisor to make the *complete* divisor 66.

5. Multiply the complete divisor by the new root

figure ($66\times6=396$) and subtract this product from the dividend 432.

6. Bring down the next period with the remainder (36) making a new dividend 3625, and proceed as before.

61. To Find the Square Root of a Decimal. Example. Find the square root of .9025.

$\overset{\frown}{.90}\ \overset{\frown}{25}(.95$ Ans.

81

Trial $9\times2=18$) 925

Complete $=175$) 925

In case of a decimal begin at the decimal point and mark off periods of *two* figures each to the right. There will be two figures in the root at the *right* of the decimal point in this example.

Notice that the square root of a decimal is larger than the decimal itself. The reason for this is easily seen if we remember that a decimal is really a fraction. .5 is really $\frac{1}{2}$. Since $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$, then the square root of $\frac{1}{4}$ must be $\frac{1}{2}$.

Example. Find the square root of 31.6969.

$\overset{\frown}{31}.\overset{\frown}{69}\ \overset{\frown}{69}(5.63$ Ans.

25

Trial $5\times2=10$) 669

Complete $=106$) 636

Trial $56\times2=112$) 3369

Complete $=1123$) 3369

In finding the square root of a whole number and a decimal we begin at the decimal point and mark off periods to the *right*, in this case *two*, also periods to the *left*, in this case *one*, making *three* periods in all.

PROBLEMS

Find the square root of the following numbers:

1. 784.
2. 8464.
3. 49,729.
4. 256,036.
5. 874,225.
6. 7832.25.
7. 98.4064.
8. 3.8416.
9. 3124.81.
10. .616225

62. The Use of Significant Figures. 1. *Reading a Machinist's Rule.* Every machinist has had the experience of measuring a piece of work that did not come exactly on a line of division on the rule he was using at the time. There is therefore some doubt in regard to what he shall call the measurement. Measurements with a rule are generally accurate enough if they come within $\frac{1}{64}$ in. of being correct. Most machinists are satisfied if they state the length of a piece of work as so many inches and sixteenths, and a "full" or a "scant" sixty-fourth.

In order to be clear on the subject, let us consider the

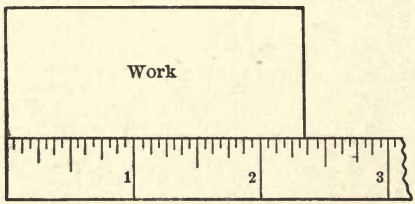


FIG. 86.—Use of Machinist's Rule.

case given in Fig. 86. Here the divisions on the scale are in sixteenths, and there are two whole inches, five-sixteenths, and a *little over*. It is the little over that cannot be measured accurately with the rule; the best we can do is to *estimate* what is beyond the five-sixteenths. To do this, divide the space between the 16th marks roughly

by eye into four parts, each of which will be $\frac{1}{64}$ in., and then estimate how many of these parts come nearest to the right measurement. In this case it is $\frac{3}{64}$, so the total measurement is 2 ins.$+\frac{5}{16}$ in.$+\frac{3}{64}$ in. Since the measurement has been taken down to 64ths, it is better to write this $2\frac{27}{64}$ ins. We must remember, however, that the last three of the 64ths were only *estimated* and are not absolutely accurate. In measuring with a rule divided into sixteenths our figures are *significant* only down to 16ths and the 64ths are *estimated.* It is important, however, to write the estimated figures, for they show how carefully the measurements have been made.

If several measurements are taken with the same rule, they should always be written in the same way. If we are looking for 64ths, 2 ins. should be written $2\frac{0}{64}$ ins., $2\frac{1}{8}$ ins. should be written $2\frac{8}{64}$ ins., $2\frac{5}{16}$ ins. should be written $2\frac{20}{64}$ ins., and so on. Measurements must sometimes be made in this way for accuracy when it is not possible to use a micrometer.

2. *Significant Figures with a Micrometer.* When readings are taken on the micrometer caliper as shown in Fig. 87, it will be remembered that the finest divisions on the sleeve are thousandths of inches. In the case shown, the mark to be read comes about half way between 17 and 18 thousandths. The readings down to thousands are *significant*, but beyond that we can only *estimate.* In this case we estimate the mark to be $\frac{1}{2}$ a thousand more than 17 and write it as .325 in.$+$.017$\frac{1}{2}$ in. or .3425 in.

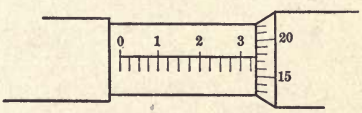


FIG. 87.

In writing measurements, .6250 in. means more than

.625 in., because it shows that we looked beyond thousandths for ten-thousandths but did not find any.

3. *Significant Figures in Multiplication and Division.* In doing problems in multiplication and division, results frequently come out with a number of figures following the decimal point. How many of these figures shall be kept for making further calculations? A good rule to follow in this case is to keep as *significant figures* as many decimal places as are found in the smaller of the two numbers multiplied or divided. Any figures beyond this may be crossed out in the result.

Examples.

```
   8.342
   2.75   (two places)
  -------
  41710
  58394
  16684
 --------
 22.940250   (keep two places)
```

```
   6.25)18.486(2.95   Divide to two places
(two places)  1250
              ----
              5986
              5625
              ----
               3610
               3125
               ----
                485
```

When the figure next to the last significant figure is 5 or more than 5, call the last significant figure 1 more, thus:

86.54625 should be written 86.55

but 86.54325 should be written 86.54.

4. *Significant Figures in Addition and Subtraction.* In addition and subtraction the numbers are all carried out to as many orders of decimals as there are in the number which has the largest number of orders. Thus in the example given below the numbers are all carried out to thousandths because two of the numbers in the column have significant figures out to thousandths.

Example.

82.006
83.200
55.723
654.865
2.780
750.000

1628.574 Ans.

5. *Significant Figures in the Use of Constants.* In using constants such as 3.1416, .7854, etc., it is a good plan to take one more decimal place in the constant than you are using in your measurements. The following example will show how little difference is made by crossing out the last part of the constant.

Example. Find the circumference of a gear blank $4\frac{1}{2}$ ins. in diameter.

Solution.

(1)
3.1416
4.5
157080
125664
14.13720 ins. Ans.

(2)
3.14
4.5
1570
1256
14.130 ins. Ans.

Multiplying by 3.14 instead of 3.1416 brings the result out only about .007 in. smaller, which does not count for much in a measurement made only down to tenths of inches in the first place.

63. The right-angled triangle. A triangle is called a **right triangle** when one of its angles is a right angle.

In a right triangle, the side opposite the right angle is called the **hypotenuse,** while the other two sides are called **legs.**

The relation of the sides of a right-angled triangle is one of most useful rules known in the calculation of distances. The rule states that: "*the square on the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides.*" If squares are drawn on the three sides of the right triangle as in Fig. 88, this fact becomes evident. By means of this rule we are able to calculate the length

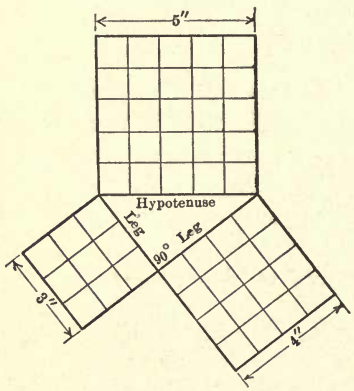


Fig. 88.

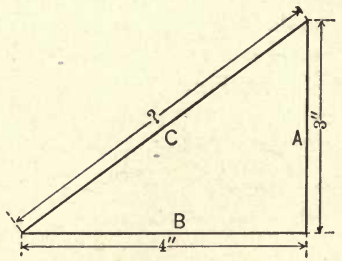


Fig. 89.

of the third side of a right triangle if the lengths of the other sides are known.

Example. The lengths of the sides of a right triangle are 3 ins. and 4 ins., Fig. 89. What is the length of the hypotenuse?

Solution. $3^2=9$ (one side) and $4^2=16$ (other side). Then $3^2+4^2=9+16=25$ (sum of squares of 2 sides). But 25 equals the *square* of the hypotenuse by the rule, therefore the square root of 25 will be the length of the hypotenuse, which is 5.

If we call one side of the triangle **A**, the other side **B**, and the hypotenuse **C**, as in Fig. 89, the rule may be conveniently written as follows:

A^2	+	B^2	=	C^2
(square of side A)		(square of side B)		(square of hyp.)

A combination of the numbers 3, 4, 5, or the multiples of these numbers will make the sides come out evenly in calculating. Thus side 3 and side 4 will give hypotenuse 5; doubling these values, side 6 and side 8 will give hypotenuse 10; side 12 and side 16 will give hypotenuse 20, and so on.

PROBLEMS

1. The sides of a right-angled triangle are 24 ft. and 32 ft. respectively. What is the length of the hypotenuse?
2. The hypotenuse of a right triangle is 15 ft., while one side is 9 ft. Calculate the length of the other side.
3. The length of one side of a right triangle is 20 ft., and the hypotenuse is 25 ft. Find the length of the other side.
4. Draw a square measuring 3 ins. on each side and calculate the length of its diagonal.
5. The diagonal of a square measures 7.0711 ins. Calculate the length of one side of the square.

64. Use of the Right Triangle in Laying out Right Angles. It frequently happens in erecting machinery that one line must be laid out at right angles with another. To do this with any degree of accuracy with a steel square would be out of the question when the work is done on

a large scale. The method shown in Fig. 90 will always give good results. In the figure, the line **AB** shows the line of the engine, and it is desired to run a line of shafting at right angles to this line. Any multiple of the combination 3, 4, 5 may be used to lay out a right-angled triangle which will give the right angle desired. Suppose we take the combination 6, 8, 10. On the line of the engine bed lay off a distance of 8 ft. from **A** to **B**. With **A** as a center and a radius of 6 ft. in the direction of **AC** describe an arc at **C**. From **B** as a center with a radius equal to 10 ft., describe an arc cutting **AC** at **C**. Joining points **A**, **B**, and **C** will give a right triangle with a right angle at **A**. The line **AC** will therefore be perpendicular to **AB**.

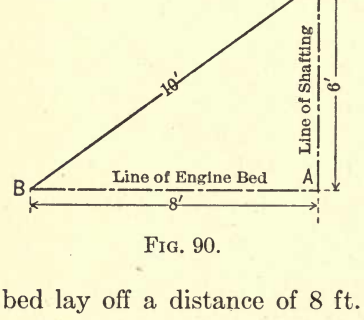


FIG. 90.

REVIEW PROBLEMS

1. Why is it more accurate to write a number as 8.50 than 8.5?
2. Find the square root of 541,696.
3. What is the 10th power of 2?
4. Should a micrometer be used on all measurements on lathe work?
5. Find the difference in calculating the circumference of a piece of work 5.6 ins. in diameter by multiplying by 3.14, then by multiplying by 3.1416.
6. What is the square root of 1?
7. What is the square root of $\frac{1}{2}$?
8. Find the square root of .5.
9. In the sketch, Fig. 91, how long must the distance AC be in order to bring the line of shafting at right angles to the line of the engine bed?

10. Find the length of the diagonal of a square measuring 1 in. on each side.

11. The foot of a ladder is 18 ft. from the base of a building, while the top of the ladder reaches a window 24 ft. from the ground. How long is the ladder?

12. A flagpole casts a shadow 120 ft. long. The distance from the top of the pole to the end of the shadow is 200 ft. How tall is the flagpole?

13. A smokestack is to be held in place by guy wires attached to the stack at a distance of 45 ft. above the ground. The wires are to be fastened in the ground at a distance of 36 ft. away from the foot of the stack. Calculate the length of a guy wire, exclusive of the amount necessary for fastening the ends.

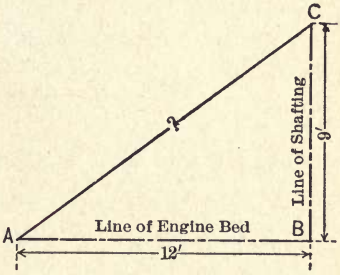


FIG. 91.

14. What is the square root of .25?

15. What is the length of the diagonal of a square measuring 6 ins. on each side?

CHAPTER IX

LATHE WORK

65. How to Center a Piece of Work. 1. *By the Use of Hermaphrodite Calipers.* The calipers are set so that the pointed leg reaches nearly to the center of the work. If the calipers are then placed at points 1, 2, 3, and 4, and an arc described at each point as shown in Fig. 92, the center of the work will be found at the middle of the figure thus drawn. If the work is very irregular, the points 1, 2, 3, and 4, must be taken at places which are as nearly equally distant as possible from the center.

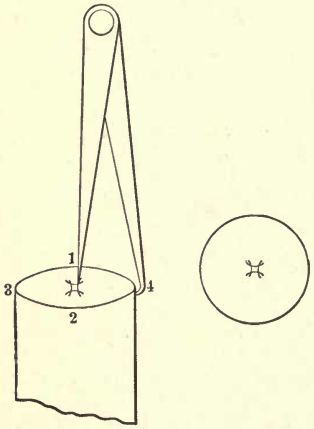


FIG. 92.

2. *By the Use of Dividers.* When the work is round and fairly true, the center may be found by placing the piece on a flat surface and drawing lines on the end as in Fig. 93. The dividers are first set to a distance which is a little more than half the diameter of the work. The work is given a quarter turn after each line is drawn until there are four lines cutting each other. The

center of the figure thus made will be the center of the work.

3. *By the Use of the Cup Center.* For small-sized round work that is fairly true and the end square, the method of Fig. 94 is rapid and accurate. The end of the piece fits into a tapering cup which automatically brings the bottom in line with the top, which fits into a second tapering cup.

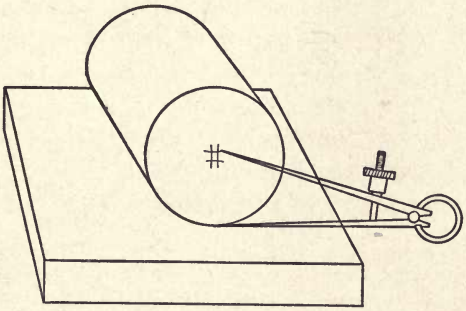


FIG. 93.

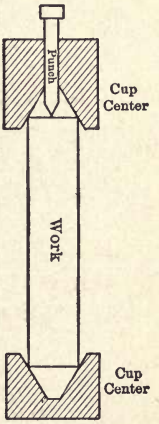


FIG. 94.

A movable prick punch is placed at the center of the cup at the top. When thus adjusted all that is necessary is to give the prick punch a light blow and the center will be marked.

If the center mark is small there is danger that the drill will not start in the right place. After a center mark is found to be in the right place it should be made quite deep by a heavy blow of a hammer vertically downward. Center marks that are slightly out of place may be moved to one side by slanting a prick punch in the direction that

the mark is to be moved and striking a blow with a hammer. The center mark should always be deepened after it is moved over.

66. Cutting Speed. The rate at which a tool passes over the work is known as *cutting speed.*

Take for example a planer tool passing over the surface of a piece of cast iron, Fig. 95. The cutting speed is generally reckoned in feet per minute, say 40 ft. per minute for cast iron. Cutting speeds have a wide variation according to the nature of the metal, the angle of the tool, the depth of cut, etc. The speed should always be as great as the tool will stand without getting too hot or wearing away too rapidly. Roughing cuts should be run at slower speed than

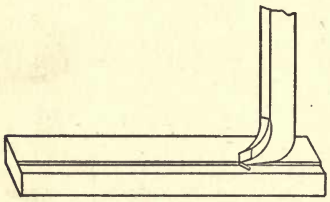


FIG. 95.—Planer Tool on Cast Iron.

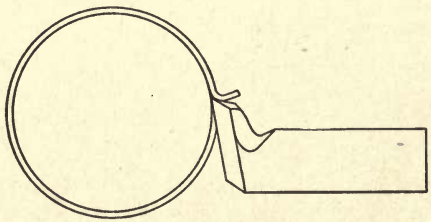


FIG. 96.

finishing cuts, because there is more strain on the tool with a heavy cut.

67. To Find the Cutting Speed for a Lathe. Suppose a piece of work is being turned in a lathe. It is easily seen, Fig. 96, that the tool passes over the whole circumference of

the work in one complete revolution. Suppose the piece of work is 6 ins. in diameter. The circumference would then be $6 \times 3\frac{1}{7} = 18\frac{6}{7}$ ins.

Reducing to feet by dividing by 12

$$18\frac{6}{7} \div 12 = \frac{132}{7} \times \frac{1}{12} = \frac{11}{7} = 1\frac{4}{7} \text{ ft.}$$

for one revolution.

If the lathe is making 28 revolutions per minute, then

$1\frac{4}{7} \times 28 = 44$ ft. per minute, cutting speed. Ans.

The cutting speed of a lathe may therefore be found by:

Circumference in feet × *number of rev. per minute*

As a piece of large diameter has a large circumference it is evident that the larger piece should have less revolutions per minute in order to avoid having too high a cutting speed for the tool.

68. To Find the Number of Revolutions for a Given Cutting Speed. Example. A cast-iron gear blank 4 ins. in diameter is to be turned in a lathe. How many revolutions per minute should the lathe run to give a cutting speed of 40 ft. per minute?

Solution.

$$\text{Circumference} = 4 \times 3\tfrac{1}{7} = 12\tfrac{4}{7} \text{ ins.},$$

$$12\tfrac{4}{7} \text{ ins.} \div 12 = 1\tfrac{1}{21} \text{ ft.}$$

In *one* revolution the tool would travel $1\frac{1}{21}$ ft. over the work. In order to cut 40 ft. per minute the work must turn as many times as $1\frac{1}{21}$ is contained in 40.

$$40 \div 1\tfrac{1}{21} = 38 + \text{rev.} \qquad \text{Ans.}$$

PROBLEMS

1. A piece of cast iron $\frac{7}{8}$ in. in diameter is turned in a lathe at 168 R.P.M. Find the cutting speed.

2. What should be the number of revolutions per minute to turn a gear blank 4.5 ins. in diameter at a cutting speed of 42 ft. per minute?

3. A piece of tool steel $\frac{3}{4}$ in. in diameter is turned in a lathe. How many R.P.M. should be taken to give a cutting speed of 30 ft. per minute?

4. In turning a cast-iron pulley 14 ins. in diameter, how many R.P.M. should be taken to give a cutting speed of 46 ft. per minute?

5. Calculate the cutting speed in turning a brass rod $1\frac{1}{4}$ in. in diameter at 252 R.P.M.

69. Cutting Feed. *Cutting feed* or *feed* is the amount of side movement of the tool along the work in one revolution. For example, the tool may move sidewise along the work $\frac{1}{20}$ in. while the lathe is making one revolution. Feed is often stated as the number of revolutions to *one inch* of sidewise travel of the tool. In the case just given, the feed would then be stated as 20, meaning 20 revolutions to 1 in. of travel of the tool, obviously the same thing as $\frac{1}{20}$ in. to one revolution.

Example. With a feed of 25, how many revolutions will it take to finish a cut on a piece of tool steel 8 ins. long?

Solution. $25 \times 8 = 200$ revolutions. Ans.

70. To Find the Time Necessary to Take a Cut. Example. How much time will be needed to take one cut over a piece of work 16 ins. long and 2 ins. diameter, with a cutting speed of 28 ft. per minute and a feed of 25?

Solution. $2 \times 3.1416 = 6.2832$ ins. $6.2832 \div 12 = .5236$ ft. circum. $28 \div .5236 = 53.47$ R.P.M. needed for cutting speed.

With a feed of 25; $25 \times 16 = 400$ rev. needed to finish.

Since there are 53.47 revolutions in *one* minute, to get 300 revolutions would take $400 \div 53.47 = 7.4+$ minutes.

Ans.

A few calculations like the above will show how easy it is to lose much time in the shop by not operating machines at the proper speed and feed. There is an economic point for different kinds of work. Machines should be worked to their capacity for all kinds of work, but not to a point where the wear on the machine will offset a gain in the time of doing the work.

The general rule is *slow speed* and *heavy feed* for roughing cuts; *fast speed* and *light feed* for finishing cuts. So many factors enter into the regulation of speed and feed that it is almost impossible to state a definite amount for each. The following table will give a fair idea of the amounts to be used under ordinary conditions:

SPEEDS AND FEEDS FOR FINISHING

	Cast Iron.		Wrought-iron Machine Steel		Tool Steel.		Brass.	
	Speed.	Feed.	Speed.	Feed.	Speed.	Feed.	Speed	Feed.
Carbon tool steel...	40	16	30	20	25	25	100	20
High-speed steel....	60	12	50	16	40	20	160	20
	Dry		Lubricated		Lubricated		Dry	

Speeds are given in feet per minute.

Feeds are given as the number of revolutions to 1 in. of sidewise travel of the tool.

For roughing cuts the speeds would be reduced about 25% and the feed made as heavy as the tool will stand.

PROBLEMS

1. How many revolutions will be necessary to take one cut over a shaft 10 ft. long with a feed of 20?

2. A shaft 6 ft. long is being turned at the rate of 50 R.P.M. With a feed of 20, how long will it take for a complete cut, allowing one minute for changing the lathe dog from one end to the other?

3. A piece of work is being turned at the rate of 40 R.P.M. With a feed of 16, how far will the tool travel in one hour?

4. How long will it take for one cut over the face of a pulley which is 36 ins. in diameter and 10 ins. face, with a cutting speed of 30 ft. per minute and a feed of 14?

5. A piece of work $1\frac{1}{4}$ in. in diameter is turned at a cutting speed of 20 ft. per minute, with a feed of 18. How many inches will be turned in 15 minutes?

71. Taper Turning. **Taper** is the difference in diameter for a unit length of a conical piece of work. It is usually stated by giving the *difference* in diameter for *one foot* of length, as .6 in. per foot, $\frac{1}{2}$ in. per foot.

If the taper is .6 in. per foot, it means that on a piece

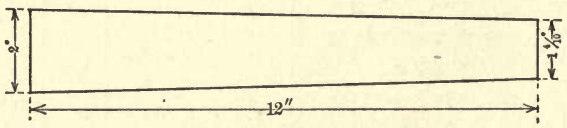


FIG. 97.

of work 1 ft. long, the small end will be $\frac{6}{10}$ in. *less* in diameter than the large end, as shown in Fig. 97.

Since the taper is a *difference* in diameter, we may have the same degree of taper on pieces of different lengths and diameters. This is shown in the sketches, Fig. 98.

72. Tapers Commonly Used. 1. *The Jarno Taper* for all sizes is .6 in. per foot. In this system the number of the taper, such as No. 1, No. 2, etc., gives a means of knowing the other dimensions used on the piece of work. In each size the number of the taper is the number of

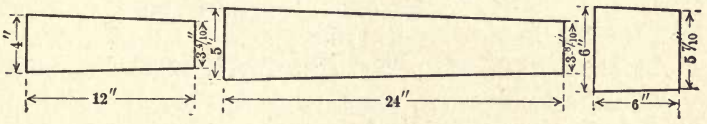


FIG. 98.

Each piece has the same taper, $\frac{6}{10}$" per ft.

eighths of an inch in diameter at the large end, the number of *tenths* of an inch in diameter at the small end, and the number of *halves* of an inch in length.

For example in a No. 6 Jarno taper, the large diameter would be $\frac{6}{8}$ in. or $\frac{3}{4}$ in., the small diameter would be $\frac{6}{10}$ in., and the length would be $\frac{6}{2}$ or 3 ins. The Jarno taper is often used on lathe centers.

2. *The Morse Taper* differs for nearly all sizes. There is no convenient way of telling dimensions. It is used chiefly on drilling machines and lathes. The Morse tapers are as follows:

No. 0 Morse taper	.625 in. per ft.
No. 1	.600
No. 2	.602
No. 3	.602
No. 4	.623
No. 5	.630
No. 6	.626

3. *The Brown and Sharpe taper* is .5 in. per foot for all sizes except No. 10, which is .5161 in. per foot. This taper is used mostly in milling machines.

4. Other tapers in use are: the *American*, $\frac{9}{16}$ in. per foot, used sometimes on drill collets; the *Power M. M.* which is about .5 in. per foot, but varies for different sizes, used on milling machine arbors; the *Sellers* taper, $\frac{3}{4}$ in. per foot, used sometimes on drill shanks.

73. Calculations for Cutting a Taper. Three methods of cutting a taper are in common use in the machine shop: (1) by offsetting the tailstock; (2) by means of the compound rest; (3) by means of the taper attachment. Only the first two of these methods will be considered here.

1. *By Offsetting the Tailstock.* When the centers of a lathe are in perfect alignment, the tool, in cutting, travels in a line parallel to the center line of the lathe, and a piece of work will be turned cylindrically. If the position of the dead center be changed by moving the tailstock slide either toward the front or back of the lathe, the work will be turned taper; small at the right-hand end if the tailstock slide is moved toward the front of the lathe, and large at the right-hand end if the tailstock slide is moved toward the back of the lathe. The more the tailstock slide is offset, the more the piece of work will be tapered.

In order to obtain a definite amount of taper it is necessary to calculate how much to offset the tailstock slide. The most important thing to remember in taper turning is that the lathe centers regulate the position of the piece to be turned, and therefore the amount of taper is determined on *the entire length of the piece between centers*, no matter what length of taper is cut on the work. Also since taper is reckoned on the *diameter* of a piece of work, and the distance between the dead center and the point

of the tool is the *radius* of the work, the amount of offset of the tailstock slide must be *one-half* the amount of the taper reckoned on the diameter. Most dimensions in machine-shop work are stated in inches, so it is convenient when making calculations to reduce the taper per foot to taper per inch.

Example. A taper of .6 in. per foot is to be turned for a distance of 5 ins. on a piece of work $8\frac{1}{2}$ ins. long. Calculate the offset of the tailstock slide.

Solution. First reduce the taper per foot to taper per inch.

$$\frac{6}{10} \div 12 = \frac{6}{10} \times \frac{1}{12} = \frac{1}{20} \text{ in. per in.}$$

Next find the amount of taper for the *whole length* of the piece between centers

$$\frac{1}{20} \times 8\tfrac{1}{2} = \frac{1}{20} \times \frac{17}{2} = \frac{17}{40} \text{ in.}$$

Take *one-half* of this because the offset is for the radius of the work.

$$\frac{17}{40} \times \frac{1}{2} = \frac{17}{80} = .213 \text{ in. (nearly) offset.} \quad \text{Ans.}$$

Remember that a taper of any length may be cut on the work after the offset is once made, so the 5 ins. length of taper does not count in the calculation of the offset.

A convenient way of getting the .213 in. offset is to clamp a tool in the tool post and bring the point of the tool against the dead center when the lathe is in alignment. The offset may now be measured by means of the dial on

the slide rest, which is generally graduated in thousandths of inches. In making this measurement be careful to look out for "lost motion" in the cross-feed of the slide rest. Even with great care it is impossible to set a taper cut with accuracy in this way so the taper should always be tested before a finishing cut is taken.

2. *By Means of the Compound Rest.* In using the compound rest to cut a taper it is very important to remember that the slide of the compound rest is set at the *complementary angle* to the angle which the taper makes with the center line of the lathe. The principle of this use of the compound rest may be shown by the cutting of the 60° angle at the end of a lathe center. This angle is not usually called a taper, but it may be cut in the same way. Angles up to 10° are usually stated as tapers, while larger angles are stated as *included angles.* The included angle at the end of the lathe center is 60°, that is, it is the whole angle at the end; the angle made with the center line will then be one-half of this angle or 30°. To cut this angle of 30°, the compound rest will be set at the complementary angle, which is $90° - 30° = 60°$.

For accuracy, the taper should be tested after taking a cut, in the same way as in the offset method.

PROBLEMS

1. Find the offset of the tailstock for tapering a piece of work 12 ins. long with a taper of .5 in. per foot.

2. Suppose, in the illustrative problem given on page 93, that the diameter of the finished work 5 ins. from the small end is $1\frac{1}{8}$ in. What is the diameter of the small end?

3. The taper on a certain piece of work is stated as .6 in. per foot. What will be the difference in diameter at two points which are 5 ins. apart?

4. A piece of tapered work measures $2\frac{3}{4}$ ins. at the large end

and $2\frac{1}{4}$ ins. at the small end. If the taper is .6 in. per foot, how long is the piece of work?

5. Calculate the offset of the tailstock for tapering a piece of work for a distance of 4 ins., when the entire length between centers is 15 ins. and the taper is .5 in. per foot.

6. A taper is stated as .5 in. per foot. If the diameter at the large end of the taper is .898 in. and the taper is $4\frac{1}{8}$ ins. long, what is the diameter at the small end?

7. The diameter at the small end of a taper shank is .356 in. If the tapered portion of the shank is $2\frac{3}{8}$ ins. long and the taper is .6 in. per foot, what is the diameter at the large end?

8. Calculate the offset of the tailstock for a taper of .625 in. on a piece of work which is $6\frac{7}{8}$ ins. long between centers.

REVIEW PROBLEMS

1. In using hermaphrodite calipers or dividers in centering a piece of work, does it make any difference if they are set a little more or a little less than the radius of the work?

2. How many revolutions per minute will it take to turn a piece of tool steel $1\frac{1}{2}$ ins. in diameter with a cutting speed of 38 ft. per minute?

3. Find the cutting speed when a shaft $2\frac{1}{2}$ ins. in diameter is turned at the rate of 70 R.P.M.

4. In the table given on page 89, calculate the speeds for

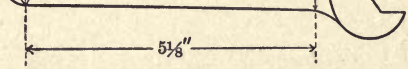


Fig. 99.

roughing cuts as 25% less than those given for finishing cuts for cast iron, wrought iron, tool steel, and brass.

5. Calculate the total taper in a piece 18 ins. long when the taper is $\frac{9}{16}$ in. per foot.

6. A taper of $\frac{1}{2}$ in. per foot is 1 in. in how many inches?

7. Find the taper per foot on the handle of the wrench given in Fig. 99.

8. Find the taper per foot on the drill shank given in Fig. 100.

9. A piece of work 6 ins. in diameter is turned at the rate of 20 R.P.M. Find the cutting speed.

10. A cast-iron pulley is machined at a cutting speed of 20 ft. per minute. If the number of revolutions is 5.09 per minute, what is the diameter of the pulley?

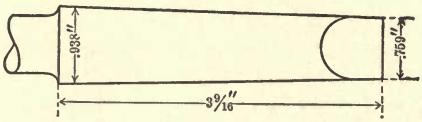


FIG. 100.

11. How many R.P.M. will it take to turn a shaft $1\frac{1}{4}$ in. in diameter so that the cutting speed will be 36 ft. per minute?

12. A piece of work 12 ins. in diameter is turned with a cutting speed of 30 ft. per minute. If the feed is 32, how long will it take the tool to travel a distance of 2 ins.?

13. On a piece of work 4 ins. in diameter, with a feed of 32,

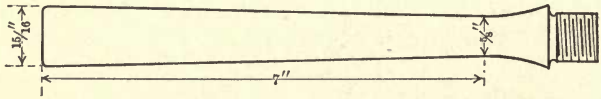


FIG. 101.

it took the tool 1 minute and 7 seconds to travel 1 in. Find the cutting speed in feet per minute.

14. Find the taper per foot on the drill holder handle given in Fig. 101.

15. A bar is tapered $\frac{3}{4}$ in. per foot. The diameters at the ends of the tapered protion are 2 and $2\frac{1}{2}$ ins. What is the length of the tapered portion?

16. A piece of work $5\frac{1}{2}$ ins. in diameter is turned with a cutting speed of 36 ft. per minute. Find the R.P.M.

17. A piece of work is turned at the rate of 5 R.P.M. with

a cutting speed of 26.18 ft. per minute. What is the diameter of the work?

18. How much should the tailstock be offset to turn a taper on a piece of work 9 ins. long, when the tapered portion is $3\frac{7}{8}$ ins. long and measures .948 in. at the large end and .778 in. at the small end.

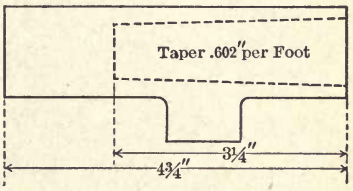


FIG. 102.

19. The piece of Fig. 102 is bored by offsetting the large end in a steady rest. If the taper is .602 in. per foot, how much must the piece be offset and in which direction?

20. In Prob. 19, if the depth of the tapered hole is $3\frac{1}{4}$ ins., and the large end of the hole is .938 in. in diameter, what is the diameter of the small end?

CHAPTER X

THREADS

74. Definitions. **A thread** is formed by cutting a uniform spiral groove around a bar, Fig. 103.

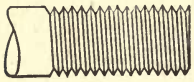


FIG. 103.

The **root** of the thread is the bottom of the groove, **A**, Fig. 104.

The **point** of the thread is the top, **B**, Fig. 104.

The **depth** of the thread is the vertical distance from the point to the root, **d**, Fig. 104.

The **diameter** of a thread is the outside diameter measured from a point on one side to the point on the opposite side. It is the same as the size of the piece before the thread was cut, **O.D.**, Fig. 104.

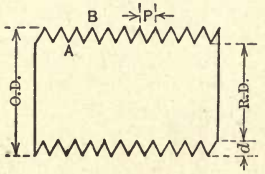


FIG. 104.

The **root diameter** is the diameter measured at the bottom of the grooves, **R.D.**, Fig. 104.

The **pitch** of a thread is the distance from the middle of the point of one thread to the middle of the point of the next, **P**, Fig. 104. It is usually stated on the basis of the number of threads in an inch; thus if there are 6 threads in an inch the pitch is $\frac{1}{6}$ in.

The **lead** of a thread is the distance that the screw moves forward in one complete turn. In a *single* thread

the lead is equal to the pitch, i.e., if the pitch is $\frac{1}{6}$ in., the screw will move forward $\frac{1}{6}$ in. in one complete turn. In a *double* thread the lead is *twice* the pitch; in a *triple* thread it is *three times* the pitch, etc.

A **right-hand thread** is the thread most commonly used. When a screw has this thread it turns in the direction of the hands of a clock when screwed into a nut.

A **left-hand thread** on a screw will cause it to turn in the opposite direction to the hands of a clock when screwed into a nut.

The threads in common use are: The Sharp " V " thread; the United States Standard thread; the Square thread; the Whitworth thread; the Acme 29° thread; the Brown and Sharpe 29° Worm thread.

75. The Sharp " V " Thread. The sharp " V " thread, Fig. 105, is sometimes used on small screws, but it is mostly a theoretical thread. It is cut by a tool sharpened to an angle of 60°. The threads should have sharp tops when cut, also a sharp groove at the bottom, but on account of the wearing of the tool in cutting, it is practically impossible to get a true sharp " V " thread.

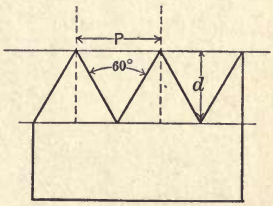


FIG. 105.

1. *To Find the Depth of a Thread.* The groove cut by a tool sharpened to an angle of 60° has the shape of a triangle with equal sides, and with equal angles at the corners, each angle being 60°.

The most important thing to be considered in cutting a thread is the depth of the cut.

Let the triangle, *A*, *B*, *C*, Fig. 106, represent the shape of the sharp " V " thread. Suppose the thread to have a pitch

of 1 in. The distance AB will therefore be 1 in. Since the triangle has equal sides, AC and BC will also be 1 in. in length. The *depth* of the thread will be equal to a line drawn from C perpendicular to AB, that is, the line CD. The line CD divides the triangle into two equal parts so that the line AB is also divided into two equal parts. Since the whole line AB is 1 in. long, the part DB will be $\frac{1}{2}$ in. Now CD was drawn perpendicular to AB, therefore the angle CDB is a right angle, and the small triangle CDB is a right-angled triangle.

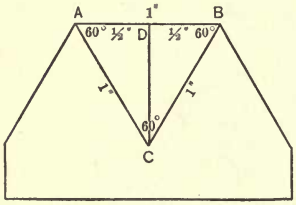


FIG. 106.

In the right-angled triangle CDB, the hypotenuse CB is 1 in. long, and the side DB is $\frac{1}{2}$ in. long, so we may easily find the value of the line CD.

By the law of the right-angled triangle (p. 80),

$$(CD)^2+(DB)^2=(BC)^2.$$

Substituting the known values of the lines we have:

$$(CD)^2+(\tfrac{1}{2}\text{ in.})^2=(1\text{ in.})^2.$$

This statement is an algebraic equation and from it we may find the value of CD. In an equation a quantity may be changed from one side to the other by changing its sign; $+(\frac{1}{2}'')^2$ changed from the left to the right side becomes $-(\frac{1}{2}'')^2$. The equation now reads:

$$(CD)^2=(1\text{ in.})^2-(\tfrac{1}{2}\text{ in.})^2,$$
$$(CD)^2=1-\tfrac{1}{4},$$
$$(CD)^2=\tfrac{3}{4}.$$

We may now take the square root of both sides of this equation, because if the same thing is done to both sides of an equation it does not change its value.

Then

$$CD = \sqrt{\tfrac{3}{4}} \quad \text{or} \quad \sqrt{.75}.$$

$CD = .866$ in., depth of thread. Ans.

2. *Depth of Sharp "V" Threads of Different Pitch.* Suppose the thread to be cut with a pitch of $\frac{1}{2}$ in. instead of 1 in. Since the thread is cut with a 60° angle on the tool, all the dimensions will be reduced in the same proportion. The sides of the triangle will now measure $\frac{1}{2}$ in. each, Fig. 107, and the *depth* of the thread will be $\frac{1}{2}$ of .866 in., or .433 in. If the pitch had been made $\frac{1}{10}$ in., the depth of the thread would have been $\frac{1}{10}$ of .866 in., or .0866 in., and so on for any pitch.

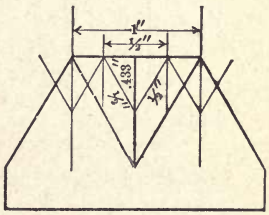


FIG. 107.

3. *Double Depth of Sharp "V" Thread.* When a thread is cut in a lathe, the tool cuts all round the piece of work so that the same depth of thread is cut on both sides of the piece. Calculations for threads are generally made on the *double depth* of the thread. In a thread of 1 in. pitch, the *double depth* will be $2 \times .866$ in., or 1.732 in. Since the depths for different pitches are in proportion, this double depth **1.732** is used as a *constant* for all sharp "V" thread calculations.

Example. Find the double depth of a sharp "V" thread with a pitch of $\frac{1}{4}$ in.

Solution. $\frac{1}{4}$ of 1.732 in. = .124 in. Ans.

The pitch of a thread is usually a fraction, so it is easier to divide by the *number of threads* per inch than to multiply by the pitch.

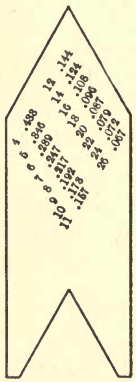


FIG. 108.

The double depth of any sharp "V" thread will then be:

$$\text{Double depth} = \frac{1.732}{\text{Number of threads per inch}}.$$

The double depths of sharp "V" threads are sometimes found on the side of a center gauge as shown in Fig. 108.

4. *To Calculate the Diameter at the Root of a Sharp "V" Thread.* **Example.** What is the diameter at the root of the thread on a bolt 1 in. in diameter, with 8 threads per inch, if the thread is a sharp "V" thread, as in Fig. 109?

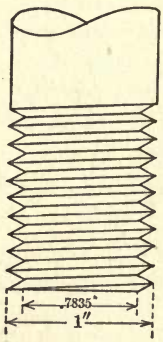


FIG. 109.

Solution. First find the double depth of the thread:

$$\frac{1.732}{8} = .2165 \text{ in. double depth.}$$

The *root* diameter will be the outside diameter less the double depth.

1 in. − .2165 in. = .7835 in., root diameter. Ans.

PROBLEMS

1. Calculate the single depth of thread for a sharp "V" thread of $\frac{1}{12}$ in. pitch.

2. Calculate the double depth of thread for a sharp "V" thread of $\frac{1}{16}$ in. pitch.

3. If the single depth of a sharp "V" thread is .0481 in., what is the pitch?

4. If the double depth of a sharp "V" thread is .0866 in., what is the pitch?

5. What is the root diameter of a sharp "V" thread on a bolt $\frac{5}{8}$ in. outside diameter with 11 threads per inch?

6. What is the root diameter of a sharp "V" thread which is $1\frac{1}{2}$ in. outside diameter with 6 threads per inch?

7. The root diameter of a sharp "V" thread is 1.0026 in. with 7 threads per inch. What is the outside diameter?

8. The root diameter of a sharp "V" thread is .5768 in. and the outside diameter is $\frac{3}{4}$ in. What is the pitch of the thread?

76. The United States Standard Thread. 1. *Form of Thread for* 1 *in. Pitch.* This thread is the one most commonly used in machine work. It has the same triangular-shaped groove as the sharp "V" thread, but is made flat at the point and at the root of the thread, Fig. 110. This flat is $\frac{1}{8}$ of the pitch in width; for a pitch of 1 in. the flat will therefore be $\frac{1}{8}$ in. *wide.* Since the depth will change in the same proportion, the amount taken from the depth will be $\frac{1}{8}$ of .866 in. at the top, and the same amount at the bottom, making in all $\frac{2}{8}$ or $\frac{1}{4}$ of .866 in., which is .2165 in. Since the whole depth is .866 in. on one side of the sharp "V" thread, the *single depth* of the United States Standard thread will be .866 in. − .2165 in.

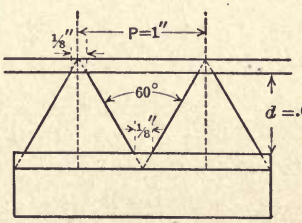


FIG. 110.—U. S. Standard Thread.

= .6495 in. The *double depth* of the U. S. Std. thread will be 2 × .6495 in., which is **1.299**. This is the *constant* for the United States Standard thread.

2. *Depth of the U. S. Std. Thread for Different Pitches.* In the same manner as in the sharp " V " thread, the *double depth* of the United States Standard thread for different pitches may be found by dividing this constant by the number of threads per inch.

$$\text{Double depth U. S. Std. thread} = \frac{1.299}{\text{number of threads per inch}}.$$

Example. What is the double depth of a United States Standard thread $\frac{3}{4}$ in. in diameter and $\frac{1}{10}$ in. pitch?

Solution. $\frac{1.299}{10} = .1299$ in. Ans.

3. *Root Diameter of the U. S. Std. Thread.* By using the constant 1.299, the root diameter of the U. S. Std. thread may be found in the same manner as for the sharp " V " thread.

77. To Find the Size Drill for a U. S. Standard Thread Tap. The root diameter of the thread is the same size as the hole to be bored for tapping the thread.

Example. What size drill should be used for a U. S. Std. $\frac{7}{8}$-in. thread tap, 9 threads per inch?

Solution: $\frac{1.299}{9} = .1443$ in., double depth of thread.

Full size of thread = $\frac{7}{8}$ in. = .875 in.

.875 in. − .1443 in. = .7307 in., size of hole. Ans.

The nearest size drill for this hole may be found by consulting a table of decimal equivalents for fractional

parts of an inch. If such a table is not at hand, the approximate size of the drill may be found as follows:

If the drill sizes run by 64ths, multiply the calculated size of hole by 64.

$$\begin{array}{r} .7307 \\ 64 \\ \hline 29228 \\ 43842 \\ \hline 46.7648 \end{array}$$

$\frac{47}{64}$ nearest size of drill. Answer.

In order to relieve strain on the tap, a drill slightly larger than the calculated size is generally used.

PROBLEMS

1. What is the single depth of a U. S. Std. thread of $\frac{1}{8}$ in. pitch?
2. What is the double depth of a U. S. Std. thread of $\frac{1}{18}$ in. pitch?
3. If the single depth of a U. S. Std. thread is .05 in., what is the pitch?
4. If the double depth of a U. S. Std. thread is .1624 in., what is the pitch?
5. Find the proper size drill to be used for a U. S. Std. $1\frac{1}{4}$-in. tap, 7 threads per inch.
6. Find the proper size drill for a U. S. Std. $\frac{9}{16}$-in. tap, 12 threads per inch.

78. Square Threads. The square thread has all its dimensions equal to each other in theory, as A, B, and C, Fig. 111. In practice the space between the threads is made a trifle larger than the width of the thread. In

the square thread there is no definite standard of pitch. Since pitch is the distance from the mid-point of one thread to the mid-point of the next, the pitch includes *one tooth* and *one space*. With a pitch of 1 in., the depth will be $\frac{1}{2}$ in., the width of tooth $\frac{1}{2}$ in., and the space $\frac{1}{2}$ in. The three dimensions will each be $\frac{1}{2}$ the pitch.

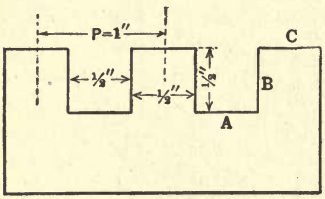


FIG. 111.—Square Thread.

79. How to Cut a Square Thread. A square thread is cut with a square-nose tool, Fig. 112, shaped something like a parting or cutting-off tool, with the exception that

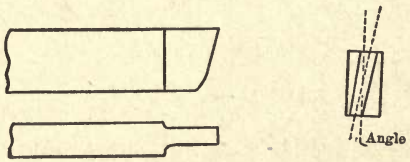


FIG. 112.

the tool must have a side rake to fit the slant of the thread.

Suppose we wish to cut a square thread with a pitch of $\frac{1}{4}$ in. If the pitch is $\frac{1}{4}$ in., the width, space and depth of the thread will each be $\frac{1}{8}$ in. The point of the square-thread tool would therefore be made $\frac{1}{8}$ in. wide at the top. The angle for side rake may be calculated as follows for a square thread $\frac{1}{4}$ in. pitch on a piece of work 1 in. in diameter.

The distance that the tool will travel around the work in one revolution will be equal to the circumference at the

root of the finished thread. The distance that the tool will travel sidewise in one revolution is equal to the pitch of the thread. In Fig. 113, the line AB is equal to the circumference at the root of the thread. At A draw AC at right angles to AB and equal to the pitch of the thread.

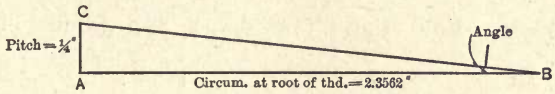


FIG. 113.

Outside Diam. = 1". Root Diam. = ¾". Circum. at Root = 2.3562".

The angle formed at B by drawing CB will be the angle of side rake. The tool should be ground somewhat thinner below the cutting point to prevent dragging on the sides of the work.

PROBLEMS

1. Calculate the root diameter of a square thread which is $2\frac{1}{2}$ ins. outside diameter, with a pitch of $\frac{1}{2}$ in.

2. Calculate the outside diameter of a square thread which is $1\frac{1}{4}$ in. root diameter, with a pitch of $\frac{5}{16}$ in.

3. Find the angle of side rake for a tool for cutting a square thread $1\frac{1}{2}$ in. outside diameter, with a pitch of $\frac{3}{8}$ in.

4. Find the angle of side rake of a tool for cutting a square thread $2\frac{1}{8}$ ins. oustide diameter, with a pitch of $\frac{5}{8}$ in.

5. If a tap is made for a square thread, should the tooth or the space of the tap be a trifle larger?

80. The Whitworth Thread. This thread is used mainly in England and Europe. The sides form an angle of 55° with each other, while the top and bottom are rounded, Fig. 114. If the thread were carried to a sharp point at the top and bottom, the rounded part would take

off roughly, $\frac{1}{6}$ at the top, and $\frac{1}{6}$ at the bottom, or $\frac{1}{3}$ in all, leaving $\frac{2}{3}$ for the depth of the thread.

The dimensions of this thread are conveniently stated in terms of the pitch; thus the depth is stated as *pitch* $\times$.64033. The radius of rounded part is also stated as *pitch* $\times$.1373.

Example. Find the pitch, depth and radius of cur-

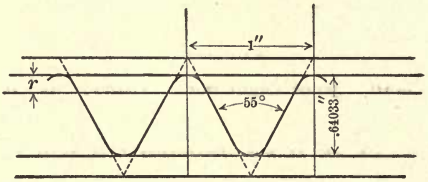


FIG. 114.—Whitworth Thread.

vature for a Whitworth Standard thread $\frac{9}{16}$ in. in diameter, 12 threads per inch.

Solution: (*a*) The pitch $= \dfrac{1}{\text{number of thds.}} = \dfrac{1}{12}''$. Ans.

(*b*) Depth $= \frac{1}{2} \times .64033 = .0534''$. Ans.

(*c*) Radius $= \frac{1}{2} \times .1373 = .0114''$. Ans.

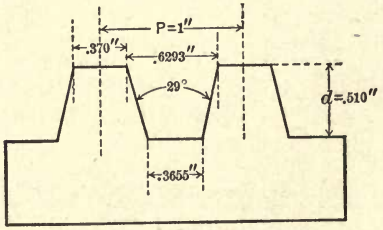


FIG. 115.—Acme Thread.

81. The Acme 29° Thread. This thread was designed to take the place of the square thread. It operates with less

friction and does away with sharp corners. The angle between the sides is 29° or $14\frac{1}{2}°$ on each side of the vertical; the included angle is also 29°, as Fig. 115.

The parts of the thread are most conveniently stated in terms of the pitch:

Depth $= \frac{1}{2}$ pitch $+ .010$ in.

Flat on top = pitch $\times .3707$.

Flat on bottom = (pitch $\times .3707$) $- .0052$ in.,

Space at top of thread = pitch $\times .6293$.

Example. Find the pitch, depth, flat on top, flat on bottom, and space at the top of a 29° Acme thread, with 8 threads per inch.

Solution.

(*a*) Pitch $= \frac{1}{8}$ in. Ans.

(*b*) Depth $= \frac{1}{16} + .010 = .0625 + .010 = .0725$ in. Ans.

(*c*) Flat on top $= \frac{1}{8} \times .3707 = .0463$ in. Ans.

(*d*) Flat on bottom $= \frac{1}{8} \times .3707 - .0052 = .0463 - .0052 =$ .0411 ins. Ans.

(*e*) Space at top $= \frac{1}{8} \times .6293 = .0787$ in. Ans.

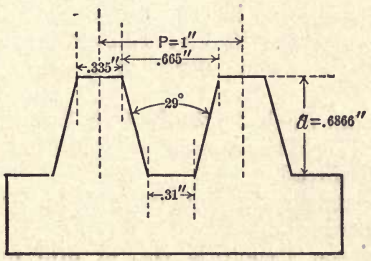


FIG. 116.—Worm Thread.

82. The 29° Worm Thread (Brown and Sharpe). The Brown and Sharpe 29° worm thread, Fig. 116, must not be confused with the Acme 29° thread, although it has the same

angle. The dimensions of this thread are stated in terms of the pitch as follows:

Depth = pitch × .6866.

Flat on top = pitch × .335.

Flat on bottom = pitch × .31.

Space at top = pitch × .665.

Example. Find the pitch, depth, flat on top, flat on bottom, and space at the top for a 29° Brown and Sharpe worm thread, 6 threads per inch.

Solution.

(*a*) Pitch = $\frac{1}{6}$ in. Ans.

(*b*) Depth = $\frac{1}{6} \times .6866 = .1144$ in. Ans.

(*c*) Flat on top = $\frac{1}{6} \times .335 = .0558$ in. Ans.

(*d*) Flat on bottom = $\frac{1}{6} \times .31 = .0517$ in. Ans.

(*e*) Space at top = $\frac{1}{6} \times .665 = .1108$ in. Ans.

REVIEW PROBLEMS

1. What is the lead of a 6-pitch single-thread screw?

2. What is the lead of an 8-pitch double-thread screw?

3. The cross-feed on a lathe has a single-thread screw of $\frac{1}{4}$ in. pitch. If the disc at the end has 25 divisions, how far will the cross-slide move when the disc is moved through one division?

4. Calculate the double depth of a sharp "V" thread of 6-pitch.

5. Calculate the single depth of a U. S. Std. thread $\frac{1}{7}$ in. pitch.

6. A square thread has a pitch of $\frac{3}{8}$ in. Find the depth, width and space of the thread.

7. Find the proper size drill for a $1\frac{3}{4}$-in. United States Std. tap, 5 threads per inch.

8. Calculate the theoretical size of drill for a sharp "V" thread $\frac{1}{4}$-in. tap, 20 threads per inch.

9. Calculate the depth, radius of curvature and root diameter for a Whitworth thread $\frac{7}{8}$ in. outside diameter, 9 threads per inch.

10. Find the flat on top, flat on bottom and space at the top of an Acme 29° thread, $\frac{3}{8}$ in. pitch.

11. In Prob. 10 find the thickness of the thread at its root.

12. If the double-depth of a sharp "V" thread is .2474 in., what is its pitch?

13. The pitch of a Brown and Sharpe 29° worm thread is $1\frac{1}{4}$ in. Find the flat on top, flat on bottom, and depth of the thread.

14. In Prob. 13 find the thickness of the thread at its root.

15. The depth of a Brown and Sharpe 29° worm thread is 1.2015 in. Find the pitch of the thread.

16. Find the root diameter of an Acme 29° thread, 4 threads per inch, when the outside diameter is $1\frac{1}{4}$ in.

17. What size drill should be used for a United States Standard $1\frac{7}{16}$-in. tap, 6-pitch?

18. What is the lead of a triple 8-pitch thread?

19. How many turns must be made with a triple 8-pitch thread screw in order to have it advance a distance of 1 inch?

20. Copy from some machinist's hand-book or catalogue a list of U. S. Std. tap sizes and numbers of threads, from $\frac{1}{4}$ in. up to 1 in. by 16ths.

CHAPTER XI

THREAD CUTTING

83. Thread Cutting. Thread cutting on a lathe is accomplished by moving the lathe carriage at a definite rate along the axis of the screw, while the tool is cutting. For *coarse* threads the carriage must move *rapidly*, and for *fine* threads *slowly* along the work.

The rate of motion of the lathe carriage is controlled by the rate of turning of the lead screw, which is in turn regulated by placing gears of different sizes on the end of the lathe, to transmit the motion of the main spindle to the lead screw. These gears can be changed at will by the operator and are called **change gears.** In order to understand how these change gears regulate the motion of the lead screw, it is necessary to know how gears of different sizes turn when meshed together.

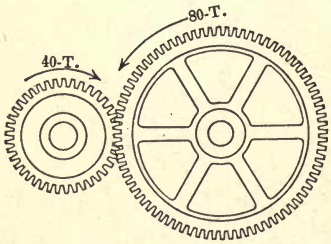


FIG. 117.—Simple Gear Train.

84. Simple Gear Train. Gears are usually known by their number of teeth, as a 40-tooth gear, an 80-tooth gear, etc. Suppose two such gears are in mesh as in Fig. 117. This arrangement is known as a **gear train.** If the small gear turns in the direction indicated by the arrow, each of its teeth will push along a corresponding tooth on

the large gear. The motion of rotation of the large gear will be in the opposite direction to that of the small one. If the small gear has 40 teeth and makes one complete turn, it will push along 40 out of the 80 teeth on the large gear. The turning of the large gear will be $\frac{40}{80}$ or $\frac{1}{2}$ a revolution. To make the large gear turn through a complete revolution, the small gear must turn twice, so the speed of the large gear is one-half that of the small one.

85. Simple-geared Lathe. A simple-geared lathe has only *one* change of speed between the spindle and lead

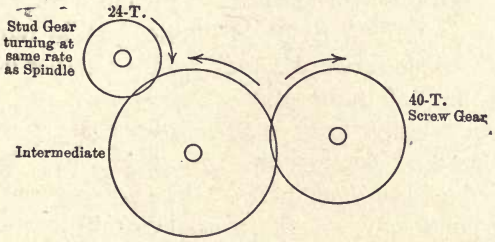


FIG. 118.—Simple-geared Lathe.

screw. The gears on this lathe are arranged for thread cutting as shown in Fig. 118. The upper gear is placed on the stud shaft, the lower gear on the lead screw, and a third gear of any convenient size, called the **intermediate**, is placed between the two. The stud shaft is placed at a distance from the lead screw in order to allow room for gears of different sizes to be placed on the stud and screw.

The intermediate does not affect the rate of turning of the lead screw, and at the same time it allows it to turn in the same direction as the stud. Suppose the stud gear has 24 teeth; as it turns, these 24 teeth will push along 24 teeth of the intermediate gear, and these in turn will push

along 24 teeth of the screw gear. The screw gear will turn $\frac{24}{40}$ or $\frac{6}{10}$ of a revolution. The same result will be obtained by leaving the intermediate out of the calculation, so we may make the calculations for change gears on a simple-geared lathe for the *stud* and *screw* only.

86. Gearing for Cutting a Single Thread. It is first necessary to find the **lead** of the machine. To do this, place gears with the same number of teeth on the stud and screw and place the intermediate in the gear train. Close the split-nut used for thread cutting on the lathe carriage, and pull the belt by hand to take up all lost motion. Next mark the position of the carriage on the lathe bed and count the *number of revolutions* of the lathe spindle required to move the carriage forward 1 *inch*. This number of revolutions is the *lead* of the machine.

Example. What gears on the stud and screw would cut a single thread of $\frac{1}{10}$-in. pitch, if the lathe has a lead of 6?

The lead of the machine is 6, so, with the same gears on the stud and screw, the lathe carriage will travel forward $\frac{1}{6}$ in. in *one revolution* of the spindle. To cut a screw with $\frac{1}{10}$-in. pitch, the lathe carriage must advance only $\frac{1}{10}$ in. for one turn of the spindle. The relation of $\frac{1}{6}$ to $\frac{1}{10}$ in. is in the same proportion as 6 to 10, or in other words the lathe carriage must travel $\frac{6}{10}$ as fast as before, since $\frac{6}{10}$ of $\frac{1}{6} = \frac{1}{10}$ in., the desired pitch. The gears which will accomplish this are a 24-tooth gear on the stud and a 40-tooth gear on the screw, as they have a ratio of $\frac{24}{40}$ or $\frac{6}{10}$.

Short Solution. The rate of travel of the carriage compared with the revolution of the spindle is a fraction (in this case $\frac{6}{10}$), whose *numerator* is always the same as the *lead* of the machine, and whose *denominator* is always the same as the *number of threads* to be cut. Start with this fraction and multiply both its numerator and denominator

by any number that will give the gears in stock on the lathe.

$$\frac{\text{Lead of machine}}{\text{Threads to be cut}} = \frac{6}{10} \qquad \frac{6}{10} \times \frac{4}{4} = \frac{24}{40'} \frac{\text{Stud gear}}{\text{Screw gear}}. \quad \text{Ans.}$$

Any two gears having the same ratio will answer just as well, as:

$$\frac{6}{10} \times \frac{6}{6} = \frac{36}{60'} \frac{\text{Stud}}{\text{Screw}} \quad \text{or} \quad \frac{6}{10} \times \frac{8}{8} = \frac{48}{80'} \frac{\text{Stud}}{\text{Screw}}. \quad \text{Ans.}$$

87. Gear Progression. Gears furnished on lathes are varied in size by adding the same number of teeth each time to the number of teeth on the gear next below in size. The number of teeth added is known as the **gear progression.** The two common gear progressions are 4 and 7. With a gear progression of 4, the smallest gear is usually 24 teeth, and proceeds 28, 32, 36, 40 and so on up to 120. With a gear progression of 7, the smallest gear is 21 teeth, proceeding 28, 35, 42, 49, and so on up to 105. A few odd-sized gears are sometimes furnished for special threads, as 69 teeth for cutting pipe threads ($11\frac{1}{2}$ threads per inch).

Since threads may be cut with any combination of gears that will give the proper relation between the stud and the lead screw, it is evidently not necessary to have all the gears in a progression from the smallest to the largest. After all combinations are calculated, only those gears which are absolutely necessary to form all the threads desired are kept and the others are dropped out of the series. On most lathes there is an index plate showing the combinations for cutting a certain range of threads with the fewest possible gears.

88. Cutting Threads with Compound Gearing. A compound-geared lathe, Fig. 119, is one where there are two changes of speed between the spindle and the lead screw. The **compound** consists of two gears, one having twice as many teeth as the other, keyed together so that they rotate at the same rate. The effect of the compound is to change the speed 2 to 1, as the small gear of the compound has only $\frac{1}{2}$ as many teeth as the large and must rotate at the same rate. With the compound in the gear

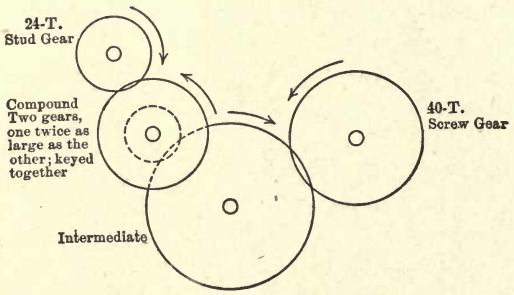


FIG. 119.—Compound-geared Lathe.

train, the lead screw will turn only $\frac{1}{2}$ as fast as it did before, and the thread cut will be $\frac{1}{2}$ of $\frac{1}{10}$ or $\frac{1}{20}$ in. pitch.

By the use of the compound, the same gears which are used to cut 8, 9, 10, etc., threads, will cut 16, 18, 20, etc. The change gears for compounding are always calculated for $\frac{1}{2}$ the number of threads required, i.e., 24 threads would be cut with gears for 12 threads, compounded.

The compound, being an extra in the gear train, is put on the lathe in the same way as the intermediate, or sometimes on a special bracket provided for it at the back of the lathe. The direction of the screw gear is changed by

having the compound in the gear train, and the tumbling gears must be reversed to get motion in the right direction.

89. Tumbling Gears or Reverse Gears on a Lathe. On the head stock of most lathes there is an arrangement of gears for giving motion in two different directions to the stud shaft. These gears are worked by a small handle placed at the left of the head stock. When the handle is placed in its top position, the train of gears in the tumbler is arranged as in Fig. 120. The motion is transmitted from gear No. 1 to No. 3, and then to No. 4, while gear No. 2

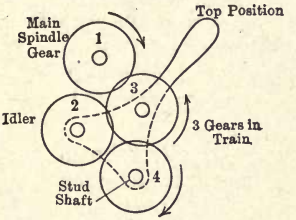


FIG. 120.—Tumbling Gears. Lever in Top Position.

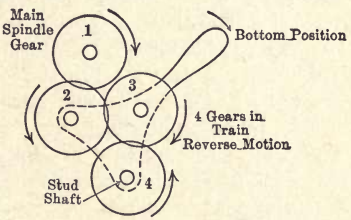


FIG. 121.—Tumbling Gears. Lever in Bottom Position.

runs as an idler. In this arrangement the stud shaft will turn in the same direction as the main spindle.

Fig. 121 shows the gears as arranged when the handle is put in its lowest position. Power is now transmitted from gear No. 1, to No. 2, to No. 3, and to No. 4. By placing gear No. 2 in the train the motion of the stud shaft is reversed and it now turns in the opposite direction to the main spindle.

90. To Cut a Right-hand Thread. Since the stud gear can be made to run in either direction by the tumbling gears, the carriage of the lathe may be moved in either direction by the motion of the lead screw. To cut a right-

hand thread the carriage is started at the tail stock and moved toward the head stock.

91. To Cut a Left-hand Thread. To cut a left-hand thread the motion of the stud gear is reversed by the tumbling gears. The thread is begun at the head stock and cut toward the tail stock.

PROBLEMS

1. A lathe has a lead of 6. If the gear progression is 4, calculate the change gears for cutting 16 threads per inch.

2. A lathe has a lead of 5. Calculate the change gears for cutting 12 threads per inch when the gear progression is 7.

3. If a lathe has a gear progression of 4, and the lead is 6, calculate the change gears for cutting $11\frac{1}{2}$ threads per inch.

4. Suppose we wish to cut 24 threads per inch on a lathe with a lead of 6 and a gear progression of 4. What gears would be used for this, by compounding?

5. Find the change gears for cutting a $\frac{1}{20}''$ pitch thread on a lathe with a lead of 6 and a gear progression of 4, by compounding.

6. Suppose a lathe has no tumbler gears and the stud revolves in the same direction as the work. How would you cut a left-hand single thread on such a lathe?

7. Suppose a lathe has a gear progression of 4 and a lead of 6. Fill out the table on page 119 for cutting the threads indicated, using as few gears as possible.

92. Planer Work. Few machinists stop to consider that cutting speed may be reckoned on a planer as well as on a lathe. It must be borne in mind, however, that on a planer the cutting is done on the forward stroke, while time is lost on the return stroke. For cutting cast iron, a planer could be run at about 40 feet per minute. The speed of a planer is regulated by the size of the pulleys on the shaft which drives it, and the speed cannot be changed after the planer is once set up. In order to take in a wide range of metals, most planers are run at about 20 ft. per

Thread.	Stud.	Screw
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		

minute on the forward movement. To save time on the return stroke, the speed is made two or three times faster

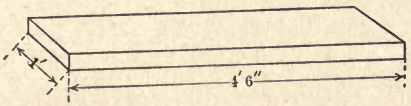


FIG. 122.

than on the forward stroke; this is known as a quick return.

The shorter we make the distance through which the platen of the planer moves, the less time will be taken for the work.

Example. How long will it take for one complete cut

on the top of the piece of work shown in Fig. 122, if the planer has a forward movement of 20 feet per minute and a return 3 times as fast, when the cut is $\frac{1}{16}$ in. wide at each stroke?

Solution. Allowing 3 ins. at each end of the work for the tool to clear, the total length of the forward movement will be 5 ft.

At 20 ft. per minute, the forward movement will be for $\frac{5}{20}$ or $\frac{1}{4}$ of a minute.

The return stroke will be $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$ of a minute.

Total time for complete stroke $\frac{1}{4} + \frac{1}{12} = \frac{4}{12}$ or $\frac{1}{3}$ of a minute.

(Actual cutting speed is 5 ft. in $\frac{1}{3}$ minute, or 15 ft. per minute.)

The feed or side motion is $\frac{1}{16}$ in. at each forward stroke, or 16 cuts to an inch. For the whole width there will be $16 \times 12 = 192$ cuts.

Time to finish $= \frac{1}{3} \times 192 = 64$ minutes $= 1$ hour 4 minutes. Ans.

(If the cuts could be made $\frac{1}{8}$ in. wide instead of $\frac{1}{16}$ in., the work could be done in half the time.)

If it is desirable to increase the cutting speed of a planer, a little calculation will show that an increase in the forward speed is more effective than increasing the return speed. There is also a loss if too much space is left at the ends of the work for clearance.

PROBLEMS

1. Find the actual cutting speed in feet per minute when a planer has a forward speed of 20 ft. per minute, and a return 2 times as fast.

2. In problem 1, suppose that the return speed is 3 times the forward speed. What change will be made in the actual cutting speed?

3. What is the actual cutting speed of a planer which has a

forward speed of 30 ft. per minute, and a return speed of 45 ft. per minute?

4. With a return speed of 3 to 1, what forward speed is necessary to obtain an actual cutting speed of 18 ft. per minute?

5. With a forward speed of 24 ft. per minute, what return speed is necessary for an actual cutting speed of 16 ft. per minute?

93. Shaper Work. 1. *Crank Shaper.* The cutting speed of a crank shaper is reckoned in an entirely different way from that of a planer. The number of strokes per minute depends upon the speed of the cone pulley which drives the shaper. The number of revolutions of the cone pulley is varied by running the belt on different steps of this pulley.

Most shapers are provided with a quick return motion. A common arrangement of the internal driving mechanism causes the back stroke to be made at about twice the speed of the forward stroke. The slow or forward movement is the cutting stroke of the tool.

Example. Find the cutting speed of a shaper running at 24 revolutions per minute, when the cut is 1 foot on each forward stroke and the back stroke is made at twice the speed of the forward stroke.

Solution. If the back stroke is twice as fast as the forward stroke, it will take $\frac{1}{3}$, and the forward stroke will take $\frac{2}{3}$ of the time of the complete stroke. Since there are 24 revolutions per minute of the crank gear, there will be 24 forward strokes in one minute. If 1 ft. of metal is cut at each of these strokes, the actual cutting speed will be 24 ft. in $\frac{2}{3}$ of a minute. In one minute the cutting speed will therefore be $24 \times \frac{3}{2}$ or 36 feet per minute. Answer.

A quicker return will give more time to the forward stroke, thus making the actual cutting speed slower. This is different from the effect in a planer, where an increase in the return speed also increases the actual cutting speed.

2. *Geared Shaper.* The cutting speed of a geared shaper does not vary with the length of stroke, but remains constant and is reckoned in the same way as a planer. Cone pulleys are sometimes provided for geared shapers, to give a change of speed for different metals.

PROBLEMS

1. Find the cutting speed of a shaper making 40 strokes a minute, if the movement of the ram is 10 ins.

2. What is the cutting speed of a shaper making 50 strokes a minute, when the length of a single stroke is 6 ins.?

3. A shaper is cutting at the rate of 40 ft. per minute. If the number of revolutions of the gear is 20, what is the length of the forward stroke of the ram?

4. The length of the forward stroke of the ram of a shaper is 14 ins. and the cutting speed is 60 ft. per minute. Find the revolutions per minute of the crank gear.

94. Drill Press Work. The cutting speeds of drills are roughly in proportion to their diameters. The speed varies with the kind of material and depth of cut. For steel, the speed at the circumference of the drill may be taken as about 30 ft. per minute; for cast iron about 35 ft. per minute; for brass about 60 ft. per minute.

Example. How many revolutions per minute should a $\frac{5}{8}$-in. drill be run when drilling cast iron?

Solution. Circumference of drill $= \frac{5}{8} \times 3\frac{1}{7} = 1.96$ in.+. A point on the circumference passes through 1.96 in. in 1 revolution. 35 ft. per minute $= 35 \times 12 = 420$ ins. per minute.

Since there are 1.96 in. for 1 revolution, the number of revolutions for 420 ins. will be $420 \div 1.96 = 214$ R.P.M. speed of drill. Ans.

A feed of 1 inch to about 100 revolutions is about right for drills.

PROBLEMS

1. Find the revolutions per minute of a $\frac{5}{16}$ in. drill when drilling cast iron at the rate of 35 ft. per minute.

2. How fast should a $\frac{3}{4}$-in. drill be run when drilling steel at the rate of 22 ft. per minute?

3. A $\frac{7}{8}$-in. drill is run at 152 R.P.M. when drilling cast iron. Find the speed at the circumference in feet per minute.

4. A $1\frac{1}{2}$-in. drill is run at 46 R.P.M. while drilling steel. Find the cutting speed in feet per minute at the circumference of the drill.

5. A drill is run at 366 R.P.M. while drilling brass at 60 ft. per minute. Calculate the size of the drill.

REVIEW PROBLEMS

1. Calculate the change gears for cutting 14 threads per inch on a lathe with a machine lead of 6 and a gear progression of 4.

2. Find the cutting speed of a shaper making 48 strokes per minute, when the length of the stroke is 8 ins.

3. What gears of the following could be used to cut 8 threads per inch on a lathe with a machine lead of 6 : 24, 30, 32, 36, 40, 48?

4. If the actual cutting speed of a planer is $13\frac{1}{3}$ ft. per minute and the forward speed is 20 ft. per minute, what is the return speed?

5. Find the revolutions per minute of a $\frac{9}{16}$-in. drill for drilling cast iron at the rate of 34 ft. per minute.

6. Calculate the time necessary for taking one complete cut, with a planer, over the surface of a piece of cast iron 36 ins. long and 8 ins. wide, when the forward speed is 24 ft. per minute, the return 2 times as fast and the feed $\frac{1}{16}$ in. at each stroke. Allow 3 ins. for clearance at the end of each stroke.

7. A shaper is cutting at the rate of 50 ft. per minute and the length of stroke is 10 ins. Find the R.P.M. of the crank gear.

8. What gears could be used, by compounding, to cut a 24-pitch screw on a lathe with a machine lead of 5 and a gear progression of 7?

9. If a lathe is arranged to cut 20 threads per inch, by compounding, with 36 gear on the stud and 60 gear on the screw,

what would be the effect of reversing the compound and making it 1 to 2?

10. A drill is making 96 revolutions per minute while drilling steel at the rate of 22 ft. per minute. Find the size of drill that should be used for this.

11. How many revolutions per minute should the crank gear of a shaper make to give a cutting speed of 40 ft. per minute, when the length of the stroke is 12 ins.?

12. Find the actual cutting speed of a planer when the forward movement is at the rate of 28 ft. per minute and the return twice as fast.

13. How many revolutions per minute should a $\frac{7}{16}$-in. drill make for drilling cast iron at the rate of 35 ft. per minute?

14. In problem 13, how long would it take the drill to go through a piece of work $1\frac{1}{2}$ ins. thick, counting a feed of 1 in. to every 100 revolutions?

15. Calculate the change gears necessary for cutting 14 threads per inch on a lathe with a machine lead of 5 and a gear progression of 7.

CHAPTER XII

SIMPLE MACHINES

ANY machine, however complicated, may be reduced in principle to one or more of the six simple machines known as the *lever, wheel and axle, pulley, inclined plane, wedge, and screw.* These six simple machines may again be reduced to only two fundamental types, the *lever* and the *inclined plane.* The wheel and axle and pulley are modified forms of the lever, while the wedge and screw are modified forms of the inclined plane.

95. The Lever. A lever may be defined as a rigid bar turning at a point called the **fulcrum,** Fig. 123. It is one of the most common of machines and is familiar as a crowbar, pinchbar, calipers, shears, etc.

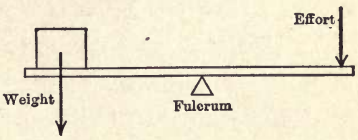


FIG. 123.—Lever.

In a **first-arrangement** lever, Fig. 124, the fulcrum is

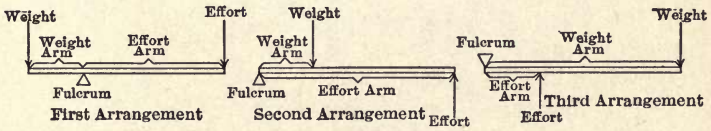


FIG. 124.

somewhere between the **effort** or force applied *to* the bar, and the **weight** or force applied *by* the bar.

In a **second-arrangement** lever, the fulcrum is at one end, the effort at the other end, and the weight somewhere between.

In a **third-arrangement** lever, the fulcrum is at one end, the weight at the other end, and the effort somewhere between.

96. How a Lever Works. 1. *First Arrangement.* Suppose a first arrangement lever to have weight, effort and fulcrum as indicated in Fig. 125. What force must be used at the place indicated to lift the weight?

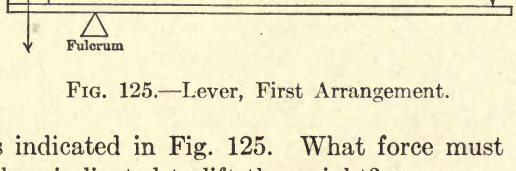


FIG. 125.—Lever, First Arrangement.

The weight of 100 lbs. acting downward has a tendency to make the bar turn in the direction of the arrow at the weight. The amount of its turning effect is known as its **moment,** and is found *by multiplying the weight by its distance from the fulcrum.* $100 \times 6 = 600$, moment. In order to make the bar balance, there must be an *equal moment* on the other side, i.e., 48 must be multiplied by a *force* great enough to make a moment of 600. This will be $600 \div 48 = 12\frac{1}{2}$ lbs. force. Ans.

The distance of the effort from the fulcrum is called the **effort arm,** and the distance of the weight from the fulcrum is called the **weight arm.**

Moments are distinguished, not from the side of the lever they are on, but by their *turning tendency.* A **right moment** always turns the lever in the direction in which the hands

of a clock turn, and a **left moment** always turns it in the opposite direction.

2. *Second Arrangement.* The same method of solution applies to all levers, only it must be remembered that the

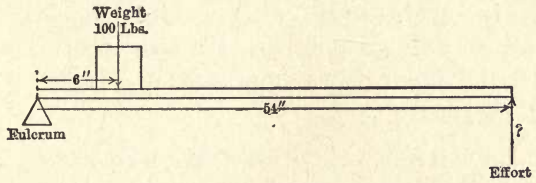


FIG. 126.—Lever; Second Arrangement.

distance of a force must always be reckoned in *a perpendicular line from the fulcrum to the point where the effort or weight is applied.* In Fig. 126, the weight arm is 6 ins., and the effort arm is 54 ins.

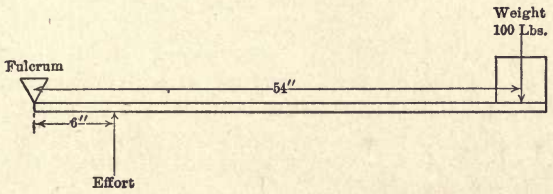



FIG. 127.—Lever; Third Arrangement.

Moment of the weight $= 100 \times 6 = 600$.

Moment of the effort (to balance) is the same, 600.

Amount of effort is $600 \div 54 = 11\frac{1}{9}$ lbs. Ans.

For a given length of bar, a second-arrangement lever is better than a first-arrangement lever, because there is a longer distance from the fulcrum to the effort.

3. *Third Arrangement.* In this case, Fig. 127, the moment of the weight is:

$$100 \times 54 = 5400.$$

The effort will be $5400 \div 6 = 900$ lbs. Ans.

There is great loss of effort in this third-arrangement lever, since the effort is greater than the weight lifted. Such a lever is useful only where a large amount of *motion* is needed. *In any simple lever:*

$$Effort \times Effort\ Arm = Weight \times Weight\ Arm.$$

PROBLEMS

NOTE. In all problems in machines, neglect friction unless the amount of friction is stated. The easiest way to solve a problem in machines is to draw a simple sketch of the forces and the distances given.

1. A man uses a piece of timber as a lever. If he is 10 ft. from the fulcrum, and the weight is placed 2 ft. on the other side of the fulcrum, how great a weight can he lift with a force of 150 lbs.?

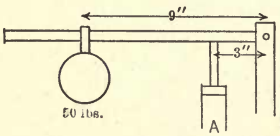


FIG. 128.—Safety Valve.

2. A safety valve is arranged as in Fig. 128. Calculate the total pressure at A that will lift the weight.

3. In problem 2, if the diameter of the valve at A is $1\frac{1}{4}$ ins., what is the pressure of the steam in the boiler per square inch?

4. In a certain machine it is desired to increase a motion from 2 to 18 ins. by means of a lever. What arrangement of lever would be used? How many times would one arm be longer than the other?

5. In a wheelbarrow, the load is centered 16 ins. from the axle, and the handles are 42 ins. from the axle. If the load is 150 lbs., what force on each handle will lift it?

6. A man uses a pinch bar to move a piece of machinery. The distance from the handle to the fulcrum of the bar is 4 ft. 9 ins., and the distance from the fulcrum to the point touching the

machinery is 3 ins. How much force is there on the machinery when 50 lbs. is applied to the handle of the bar?

7. Two men are carrying a casting weighing 200 lbs. on a bar 6 ft. long. Neglecting the weight of the bar, how much must each man lift if the weight is 2 ft. from one end, and the men are at opposite ends of the bar. (Consider one man as a fulcrum and solve as a second-arrangement lever).

8. A pump handle is arranged as a first-arrangement lever. The distance from the end of the handle to the fulcrum is 18 ins. and the piston rod is pivoted at a point 2 ins. on the other side of the fulcrum. How great a pressure will 20 lbs. at the end of the handle give on the piston rod?

9. The foot treadle shown in Fig. 129 is 16 ins. long. If the foot is placed 4 ins. from the fulcrum, with how much force must it push to exert 15 lbs. at A?

10. Design a first-arrangement lever, also a second-arrangement lever, where 10 lbs. will lift 50 lbs. in each case.

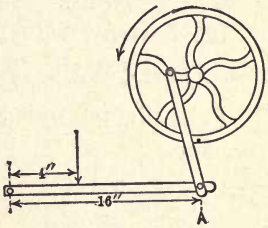


Fig. 129.—Foot Treadle.

97. The Wheel and Axle. If a large wheel and a small axle are so fastened that they turn on the same axis, the arrangement is known as a **wheel and axle,** Fig. 130. Common examples of this machine are: the hoist on a derrick, the capstan on a ship, the dumb waiter hoist, etc.

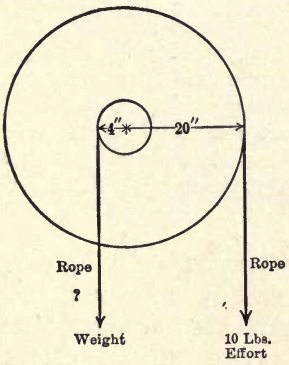


Fig. 130.—Wheel and Axle.

When the wheel makes one revolution, the effort moves downward a distance equal to the circumference of the wheel, while the weight moves upward a

smaller distance equal to the circumference of the axle. Since the radii of two circles have the same relation to each other as the circumferences of the circles, we may consider the wheel and axle as a modified lever of the first or second arrangement.

In Fig. 130, the effort is 10 lbs. and acts at a distance of 20 ins. from the axis of the wheel, while the weight acts at a distance of 4 ins. from the same axis.

Moment of the effort is $10 \times 20 = 200$; amount of weight $= 200 \div 4 = 50$ lbs. Ans.

98. Law for wheel and axle.

$$Effort \times Radius\ of\ Wheel = Weight \times Radius\ of\ Axle.$$

(The diameter or circumference would do just as well in this law.)

PROBLEMS

1. The radius of a wheel is 18 ins. and the radius of the axle is 3 ins. What weight can be supported by an effort of $16\frac{2}{3}$ lbs.?

2. An effort of 20 lbs. supports a weight of 120 lbs. on a wheel and axle. If the diameter of the axle is 8 ins., what is the diameter of the wheel?

3. The capstan of a ship has a circumference of 36 ins. The handspikes are of such a length that the men walk through a circumference of 15 ft. in turning the capstan. How much force must be exerted by each of 4 men in raising a 2000-lb. anchor?

4. Two gears are keyed together on one shaft. If the larger has a diameter of 14 ins. and the smaller a diameter of 5 ins., what force will be exerted at the smaller by a power of 20 lbs. on the larger?

99. The Pulley. The pulley consists of a grooved wheel called a **sheave** around which a rope passes. The sheave is supported in a framework called a **pulley block.** By reference to Fig. 131, it will be seen that a pulley is like a first-arrangement lever with equal arms. This is a

fixed pulley and only changes the direction of pull. If a force of 10 lbs. is applied downward at one side of the rope, a weight of 10 lbs. will be lifted upward at the other side (neglecting friction).

In Fig. 132, the lower pulley is known as a **movable pulley.** It not only turns around in the pulley block, but goes up or down at the same time. Suppose there is a pull of 10 lbs. downward at the free end of the rope. Call

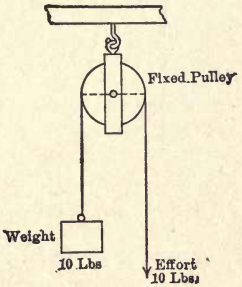


FIG. 131.—Single Pulley.

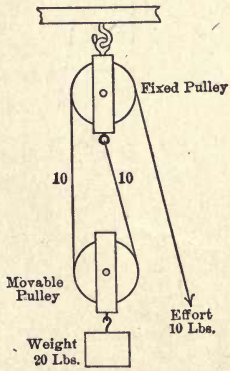


FIG. 132.—Fixed and Movable Pulleys.

this the effort. Every part of the rope will feel this pull, and there are two parts holding up the movable pulley, each with 10 lbs., so that a weight of 20 lbs. can be supported.

If the effort goes downward a distance of 1 ft., the rope will be shortened that amount. The loop of rope that goes around the movable pulley will be shortened in all 1 ft., or $\frac{1}{2}$ ft. on each side, so the movable pulley will rise only $\frac{1}{2}$ ft.

To find how much a pulley system will lift we need only count the number of parts of rope that hold up the movable pulley. **The law** is most conveniently stated as follows:

Effort $\times$ *Number of Ropes on Movable Pulley* = *Weight.*

It makes no difference whether the pulleys are placed along-

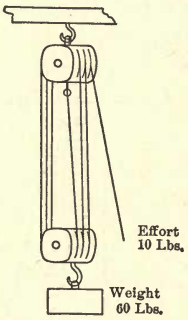


FIG. 133.—Pulleys Placed Side by Side.

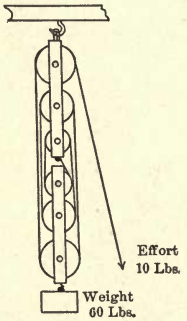


FIG. 134.—Pulleys Placed under One Another.

side of each other as in Fig. 133, or under each other as in Fig. 134; the same law holds.

PROBLEMS

1. With three fixed and two movable pulleys, how much weight can 10 lbs. support?

2. Draw a sketch of a system of pulleys that will allow 120 lbs. to be supported by 40 lbs.

3. Would it be of any advantage in a system of two fixed and two movable pulleys to have the outside rope pull upward on the movable pulleys?

4. What arrangement of fixed and movable pulleys will allow 10 lbs. to support six times its own weight?

100. Mechanical Advantage. In a lever, a small force applied to a long lever arm will lift a large weight on

a short lever arm; in a wheel and axle, a small force applied at the circumference of the wheel will exert a greater force at the circumference of the axle; in a pulley system, a small force applied to the outside rope will lift a large weight at the movable pulley block. This gain in force is known as **mechanical advantage.** The mechanical advantage of a machine may be calculated in two ways: (1) by dividing the weight lifted by the effort applied; thus if the weight is 50 lbs. and the effort 10 lbs., the mechanical advantage is $50 \div 10 = 5$; (2) by dividing one dimension of a machine by another. In a first-arrangement lever it is found by dividing the long arm by the short arm; if the long arm is 6 ft., and the short arm 2 ft., the mechanical advantage is $6 \div 2 = 3$. In a wheel and axle, the mechanical advantage is the diameter of the wheel divided by the diameter of the axle.

Many machine problems may be solved easily by using the principle of mechanical advantage. If a machine has a mechanical advantage of 5, an effort of 10 lbs. will lift 5 times as much weight, or 50 lbs. With this same mechanical advantage it would take an effort of 100 lbs. to lift a weight of 500 lbs.

101. The Inclined Plane. *Case* 1. *Force Parallel to Incline.* When a man wishes to get a barrel from the ground into a wagon, the task is made easy by rolling the barrel along a plank or pair of skids. This arrangement is known as an **inclined plane.** The simplest way of exerting the force is in a direction parallel to the incline. In Fig. 135, suppose an iron roller weighing 100 lbs. is resting on the incline. The vertical height at the end of the plane is 3 ft., and the length of the incline is 12 ft. If we measure the force, applied at the center of the roller, which will just lift the roller up the plane (neglecting friction),

it will be found to be 25 lbs. (An easy way to measure it is by a spring balance.) The 100 lbs. has the same relation to its supporting force (25 lbs.) as the length of the incline (12 ft.) has to its vertical height (3 ft.). In other words

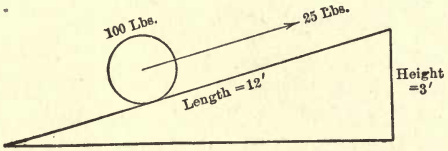


FIG. 135.—Inclined Plane.

the *mechanical advantage* of the incline is the *length of the incline divided by its vertical height.* Observe also that the effort times the length of the incline gives a product, $25 \times 12 = 300$, which is always equal to the product of the weight times the height of the incline, $100 \times 3 = 300$. The **law for an inclined plane,** when the effort acts parallel to the incline, may be stated:

Effort $\times$ *Length of plane* = *Weight* $\times$ *Height of plane.*

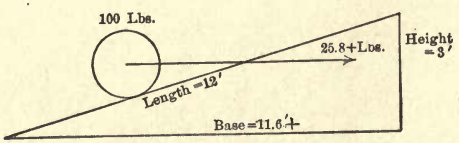


FIG. 136.—Inclined Plane; Force Parallel to Base.

Case 2. *Force Parallel to the Base of the Plane.* If the effort acts parallel to the base of the inclined plane, Fig. 136, the mechanical advantage is found by dividing the *base* of the plane by its *vertical height.* The base of the plane

is shorter than the incline, so the mechanical advantage in this case is less than in the first case. **The law for case 2** may be written:

Effort × *Base of plane* = *Weight* × *Height of plane.*

PROBLEMS

1. Skids 10 ft. long are used to roll a barrel weighing 196 lbs. from the ground to a vertical height of $2\frac{1}{2}$ ft. What force parallel to the skids will be used?

2. A 1200-lb. piece of machinery is to be lifted from the ground to a platform 2 ft. high. The greatest force that can be applied is 200 lbs. What length of inclined plane can be used?

3. A power of 250 lbs. supports 1250 lbs. on an inclined plane 10 ft. long. Find the height of the plane.

4. The base of an inclined plane is 14 ft. and the vertical height is 3 ft. How great a weight will a power of 60 lbs., acting parallel to the base, support?

5. By the law of the right-angled triangle, find the length of the base of an inclined plane, when the incline is 5 ft. and the vertical height 3 ft.

6. In problem 1, find what effort acting parallel to the base will support the weight.

102. The Wedge. The wedge may be considered as two inclined planes placed with their bases along the center line or length of the wedge. No law that is of much practical value can be given for the wedge. In Fig. 137, if we neglect friction, the wedge would lift the upper block 1 in. if driven in its whole length of 6 ins. The effort moves 6 times as far as the weight, so the

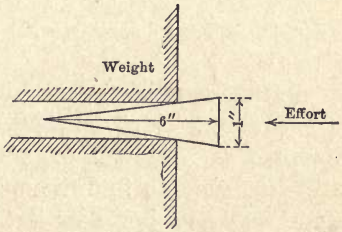


FIG. 137.—The Wedge.

mechanical advantage of the wedge is 6, or in general the *length divided by the width.* The **law for the wedge** may be stated as follows:

$$Effort \times Length = Weight \times Width.$$

Example. If a wedge is 12 ins. long and 3 ins. wide, what force will be exerted by the wedge when 100 lbs. effort is applied to the " head " of the wedge?

Solution. Mechanical advantage $= 12 \div 3 = 4$.

$100 \times 4 = 400$ lbs. exerted. Ans.

103. The Screw. A screw is like an inclined plane rolled up in the form of a cylinder. This may be shown

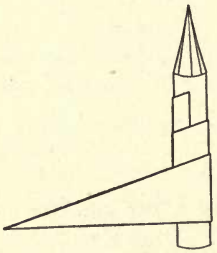


FIG. 138.—Thread of a Screw.

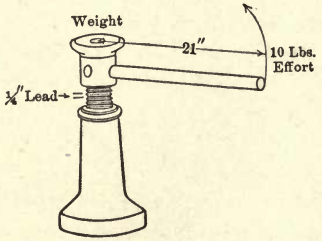


FIG. 139.—Jack Screw.

by cutting a right-angled triangle out of a piece of paper and wrapping it around a pencil as shown in Fig. 138. The thread of the screw is formed by the incline of the plane. It is not easy to find the mechanical advantage of a screw unless we know just how the effort is applied. The following case will serve as an illustration:

In the jack screw shown in Fig. 139, suppose the effort is applied at the end of the lever. In one revolution of

the screw this effort will travel the *circumference of a circle of which the lever is a radius.* The screw advances a distance equal to its lead in one revolution; in a single thread this is the same as the pitch. If the effort goes through the circumference, and the weight goes a distance equal to the lead, the mechanical advantage of the screw must be *equal to the circumference divided by the lead.* **The law** may also be stated thus:

Effort × *Circumference for the effort* = *Weight* × *Lead of screw.*

Example. With a jack screw having a lead of $\frac{1}{4}$ in. and a lever 21 ins. long, what force will be exerted by an effort of 10 lbs. applied at the end of the lever?

Solution.

Circum. for the effort = diameter $\times 3\frac{1}{7} = 42$ ins. $\times 3\frac{1}{7} = 132$ ins.
Lead of screw = $\frac{1}{4}$ in.
Mechanical advantage = $132 \div \frac{1}{4} = 132 \times \frac{4}{1} = 528$.
10 lbs. $\times 528 = 5280$ lbs. exerted. Ans.

PROBLEMS

1. A jack screw has a lever 16 ins. long and the lead of the

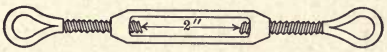


FIG. 140.—Turnbuckle.

screw is $\frac{1}{3}$ in. What force will be exerted by 20 lbs. pull at the end of the lever?

2. A vise has a 4-pitch, single-thread screw and the handle is 8 ins. long. If a man pushes with 50 lbs. at the end of the handle, what force is exerted by the screw on the jaws of the vise?

3. What would be the effect of having an 8-pitch screw instead of a 4-pitch screw in problem 2?

4. A turnbuckle, Fig. 140, has 12 threads per inch on each end.

The ends of the rods as shown in the sketch are 2 ins. apart. How far apart will the ends be after 10 complete turns in bringing the turnbuckle together?

5. If the turnbuckle in problem 4 is turned by a monkey wrench so that the effort is applied 8 ins. from the center, what force will be exerted by the turnbuckle when the force on the monkey wrench is 20 lbs.?

104. Friction. When one body moves or slides over another, the pressure between the two bodies causes a resistance to motion. This resistance to motion is known as **friction** and it increases directly with the pressure. Friction reduces the efficiency of a machine. If we make the surfaces smooth, use oil or grease on bearings, have ball or roller bearings, we diminish the friction and increase the efficiency.

Friction is sometimes an advantage, as in the case of putting sand on tracks to increase the friction so that the wheels may get a grip on the rails.

105. Mechanical Efficiency. In all our calculations of simple machines we have so far neglected the effect of friction. If we take friction into account we may reckon the mechanical efficiency of a machine. The **mechanical efficiency** of a machine is the relation between the work put into a machine and the work gotten out of it. It should not be confused with *mechanical advantage*. Mechanical efficiency is stated as a per cent. If 100 units of work are put into a machine and only 90 units of work are gotten out (the rest being lost by friction), the efficiency of the machine is $\frac{90}{100}$ or 90%.

(The meaning of the term " unit of work " is explained in Chapter XIII, page 143.)

In a lever, or wheel and axle, there is little loss from friction, but in a pulley system the loss may be from 10% to 50%. The efficiency of an inclined plane varies from

almost nothing to over 90%. A screw has less than 50% efficiency. A rough way of judging the efficiency of a machine is to see if it will go back of its own accord when the effort is removed, as for instance in a block and tackle. If it just goes back, its efficiency is somewhere around 50%; if it holds, the efficiency is less than 50%.

106. General Law of Machines. In most machines the effort moves through a large distance while the weight moves through a small distance. If the effort is multiplied by the distance through which it moves, a product will be obtained, which is equal to the product obtained by multiplying the weight by the distance through which it moves. This is known as the **general law of machines,** and may be applied to any machine, simple or compound. It is conveniently stated as follows:

$$Effort \times Distance\ of\ effort = Weight \times Distance\ of\ weight.$$

107. Compound Machines. No matter how complicated any machine may appear, when it is analyzed, it will be found to be made up of combinations of the six simple machines. The easiest way to work out the mechanical advantage of any compound machine is by the general law of machines, taking the force applied at one end of the machine as the effort and the force exerted at the other end of the machine as the weight.

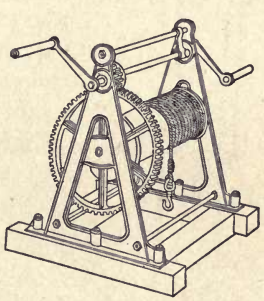


FIG. 141.—Winch.

The compound machine shown in Fig. 141 is used to raise heavy weights by hand power. Suppose the crank has a radius

of 24 ins., the barrel a radius of 6 ins., and that the small wheel has 16 teeth and the large wheel 96 teeth.

For one revolution of the large gear there will be $96 \div 16 = 6$ revolutions of the small gear, therefore the crank will turn 6 revolutions to 1 revolution of the barrel.

Circumference of the effort $= 24 \times 2 \times 3\frac{1}{7} = 150$ ins.+

Distance effort moves $= 150 \times 6$ (rev.) $= 900$ ins.

Distance weight moves = circum. of barrel $= 6 \times 2 \times 3\frac{1}{7} =$ 37 ins.+

Mechanical advantage $= 900 \div 37 = 24+$

If 50 lbs. is applied at the crank it will support:

$50 \times 24 = 1200$ lbs. at the barrel (neglecting friction).

50 lbs. at both cranks would support $1200 \times 2 = 2400$ lbs.

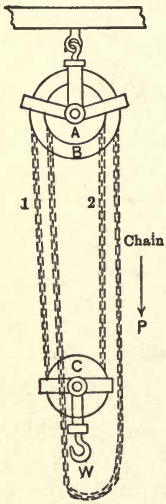


FIG. 142.—Differential Pulley.

108. The Differential Pulley. A machine that is widely used for lifting heavy weights by hand is the differential pulley or hoist, Fig. 142. It is cheap but not efficient. Two fixed pulleys, A and B, of different diameters, are fastened together and rotate as one, while a single movable pulley, C, is suspended below. An endless chain passes over each fixed pulley, and one loop of the chain passes around the movable pulley.

There are lugs in the pulley grooves to prevent the chain from slipping. If the chain is pulled in the direction of the arrow until the large fixed pulley B turns around once, the part of the chain marked 1 will be shortened a distance equal to the circumference of the large fixed pulley; but at the same time the small fixed pulley A lets the part of the chain marked 2, down a distance equal

to its circumference. The actual shortening of the loop 1, 2, will be equal to the difference in the circumferences of the two pulleys. Pulley C will, however, rise only $\frac{1}{2}$ this amount, because there are two parts to the loop, and the weight will go up with C.

Suppose the circumference of the large fixed pulley is 36 ins., and the circumference of the small fixed pulley 30 ins.

Distance effort moves = circumference of large pulley = 36 ins.
Dist. weight moves = $\frac{1}{2}$ difference of circum. = $\frac{1}{2}(36-30) = 3$ ins.
Mechanical advantage = $36 \div 3 = 12$.
50 lbs. will support $50 \times 12 = 600$ lbs. (neglecting fric.). Ans.

Suppose friction is so great that the efficiency of the differential pulley is $33\frac{1}{3}\%$. Instead of lifting 600 lbs. with 50 lbs., only $33\frac{1}{3}\%$ of 600 or 200 lbs. would be lifted.

REVIEW PROBLEMS

(In all problems neglect friction unless its amount is stated.)

1. The radius of a wheel is 20 ins. while the radius of the axle is 4 ins. What weight can be supported by an effort of 50 lbs.?

2. Sketch a system of 2 fixed and 2 movable pulleys and calculate what effort will raise 500 lbs. if friction increases the load 25%.

3. Show by sketch a second-arrangement lever on a pump so that 50 lbs. applied at the handle will exert 400 lbs. on the piston placed 6 ins. from the fulcrum.

4. A weight of 900 lbs. is to be lifted with 3 fixed and 3 movable pulleys. If each man can pull with a force of 125 lbs., how many men will be necessary?

5. A differential hoist has fixed pulleys 8 and 5 ins. in diameter. What theoretical effort will lift 352 lbs.?

6. If the efficiency of the differential hoist in problem 5 is 35%, what effort would be required?

7. A 1500-lb. safe is to be moved from the sidewalk into a truck

4 ft. above the ground. Skids 12 ft. long are used. What theoretical force will move the safe?

8. A jackscrew with a lead of $\frac{1}{2}$ in. has a handle 20 ins. long. How much pressure will a pull of 60 lbs. at the end of the handle give at the end of the screw?

9. A lathe test indicator has a short arm $\frac{3}{4}$ in. long, and a pointer 12 ins. long. How much will the short end move when the end of the pointer moves $\frac{1}{8}$ in.?

10. How many jackscrews with levers 18 ins. long and a lead of $\frac{1}{3}$ in. will be required to raise a 40-ton building, if the pull on each lever is 60 lbs.?

11. Design a differential hoist so that one man by exerting 100 lbs. may lift a ton. (Neglect friction.)

12. A windlass has a barrel 8 ins. in diameter and a crank 20 ins. long. A rope extends from the windlass to 3 fixed and 2 movable pulleys. If an effort of 50 lbs. is exerted on the crank, what weight can be lifted at the movable pulleys?

13. What is the total mechanical advantage in problem 12?

14. If the efficiency of the machine in problem 12 is 40%, what weight can be lifted?

CHAPTER XIII

WORK, POWER, RATIO AND PROPORTION, GEAR RATIOS, PULLEYS, BELTING

109. Work. Work is done when a force acts on a body in such a way as to move it. A man may put forth great force in trying to move a body, but unless the body is actually moved no work is done. There are two things which must be reckoned in work, *force* and *distance*. Work is measured by a unit called a **foot-pound.** It is the work done when a force of 1 lb. moves a body through a distance of 1 ft. If 1 lb. of force moves a body through 2 ft. there will be 2 ft.-lbs. of work done. A force of 25 lbs. acting through a distance of 10 ft. will do 25×10 or 250 ft.-lbs. of work.

110. Power. Power is the rate of doing work, that is, it is the number of foot-pounds of work that can be done in *one second.*

Example. What power is used when a force of 25 lbs. moves a body 10 ft. in 2 seconds?

Solution. Total work done $= 25 \times 10 = 250$ ft.-lbs.

$250 \div 2 = 125$ ft.-lbs. in 1 second. Ans.

Horse-power is the ability to do 550 ft.-lbs. of work in 1 second. Engineers commonly use horse-power on the time basis of a minute. $550 \times 60 = 33{,}000$ *ft.-lbs. per minute.*

Example. If an engine lifts 6600 lbs. 40 ft. high in 2 minutes, what horse-power is exerted?

Solution. Total work = $6600 \times 40 = 264{,}000$ ft.-lbs.
Work in 1 minute = $264{,}000 \div 2 = 132{,}000$ ft.-lbs.
Horse-power = $132{,}000 \div 33{,}000 = 4$. Ans.

111. Horse-power of a Steam Engine. 1. *Indicated Horse-power.* When the horse-power of an engine is spoken of, the indicated horse-power is generally meant.

Example. Find the horse-power of a steam engine with a cylinder 18 ins. in diameter, 36-in. stroke, running at 70 revolutions per minute, with an average pressure of 75 lbs. per square inch on the piston.

Solution. Total force on piston = lbs. per sq.in. $\times$ area.
$75 \times (18)^2 \times .7854 = 19{,}085.25$ lbs. force.
Distance force moves = stroke $\times$ *twice* (forward and back) the R.P.M. = $\frac{3}{1}\frac{6}{2} \times 70 \times 2 = 420$ ft., distance moved.
Ft.-lbs. of work = $19{,}085.25 \times 420 = 8{,}015{,}805$.

$$\text{Horse-power} = \frac{8{,}015{,}805}{33{,}000} = 242.+ \qquad \text{Ans.}$$

The pressure per square inch is found by an indicator card. When this is not done, the average pressure may be taken as one-half the boiler pressure.

2. *Brake Horse-power.* The brake horse-power of an engine is the power actually delivered by the engine. It may be determined by putting a brake on the rim of the fly-wheel and letting the engine run against this brake.

The " Prony Brake " shown in Fig. 143, consists of a metal band with a number of wooden blocks fastened to it, resting on the rim of the fly-wheel. The pressure of the blocks may be regulated by tightening the screw at *B*. Friction between the blocks and the rim of the fly-wheel will make the brake tend to turn in the direction of the

arrow. The force of turning may be measured by the platform scale at *A*. Allowance is made for the weight of the brake itself by finding the pressure on the scale when the brake is loose.

The distance from the center of the fly-wheel to where the brake rests on the platform is the radius of a circle

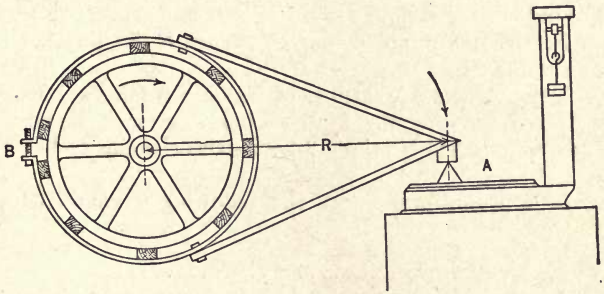


FIG. 143.—Prony Brake.

through which the force would act, if allowed. This distance is the *arm* of the force.

Example. In a "Prony Brake" test the force on the scale was 30 lbs., the distance from the center of the fly-wheel to the point resting on the scale was $3\frac{1}{2}$ ft., and the wheel turned at 400 revolutions per minute. Find the brake horse-power.

Solution. Work in one revolution = force × circum. of arm

$$30 \times 3\tfrac{1}{2} \times 2 \times 3.1416 = 659.7 \text{ ft.-lbs.}$$

Work done in 1 minute = $659.7 \times 400 = 263{,}880$ ft.-lbs.

$$\text{Horse-power} = \frac{263{,}880}{33{,}000} = 7.99+ \qquad \text{Ans.}$$

PROBLEMS

1. How much work is done when an elevator weighing 1200 lbs. is lifted 40 ft. high?

2. Find the power used when 600 lbs. is lifted 20 ft. in 10 seconds.

3. How many foot-pounds of work per minute may be obtained from an engine delivering 15 horse-power?

4. Find the horse-power of a 10 by 24-in. steam engine running at 90 R.P.M., with an average pressure of 75 lbs. per square inch on the piston.

5. A pump lifts 12,000 lbs. of water per minute into a tank 100 ft. above the ground. Find the horse-power of the pump.

6. How long will it take a 2 horse-power engine to lift 550 lbs. 60 ft. high?

7. In a Prony-brake test the following measurements were obtained:

Radius of brake	$3\frac{1}{4}$ ft.
Pressure on scales	34 lbs.
R.P.M. of fly-wheel	420

Find the horse-power developed.

8. How many pounds can an 8 horse-power engine lift 40 ft. high in 3 minutes?

112. Ratio and Proportion. A **ratio** is the relation between two quantities of the *same kind.* If a piece of iron weighs 50 lbs. and another piece weighs 100 lbs., the second is twice as heavy as the first, or the ratio of their weights is 1 to 2. Ratios may be written in several different ways, as, 1 to 2, 1 : 2, $\frac{1}{2}$, $1 \div 2$. The most common way is 1 : 2.

A proportion is an equality of two ratios; as, $1 : 2 = 2 : 4$. In a proportion there are always four numbers. These need not all be of the same kind, but each ratio must be made up of quantities of the same kind. 1 lb. : 2 lbs. = 2 ft. : 4 ft. is a true proportion.

In any proportion the two *outside* numbers are called

the **extremes,** and the two *inside* numbers are called the **means.** In a proportion, *the product of the means is equal to the product of the extremes,* as:

$$1 : 2 = 2 : 4$$

Then $$2 \times 2 = 1 \times 4,$$

$$4 = 4.$$

Proportion gives us a simple way of solving many questions.

A direct proportion is one whose ratios vary in the *same order.*

Example. If 15 castings cost \$24, what will 40 similar castings cost?

Solution. This is a direct proportion because the cost of the castings increases directly with their number.

$$15 : 40 = 24 : x.$$

The "x" stands for an unknown quantity and may be handled just like a number.

Product of extremes: $15 \times x = 15x$.

Product of means: $40 \times 24 = 960$.

Since the product of the means = product of the extremes, then $15x = 960$.

Dividing each side by 15,

$x = 64$, or \$64 cost of castings. Ans.

One of the most common faults in handling a propor-

tion is to get the ratios in the wrong order. In the proportion

$$\begin{array}{cccccc} 15 & : & 40 & = & 24 & : & x \\ (1) & & (2) & & (3) & & (4) \end{array}$$

if we number the terms 1, 2, 3, 4, from left to right, the correct order will be: (2) is larger than (1), so (4) must be larger than (3). If (2) had been less than (1), then (4) would have been less than (3). Always look carefully at the answer and see if it is reasonable; if not, one of the ratios must be in the wrong order.

An indirect or inverse proportion is one where the ratios vary in the *opposite order*. Suppose two gears are running together as in Fig. 144; if we consider the number of revolutions and the number of teeth; the large gear turns slowly but has a greater number of teeth, while the small gear turns swiftly and has a less number of teeth. The number of revolutions of each gear varies in the *opposite way* from the number of teeth on the gear.

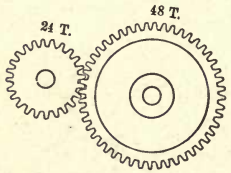


FIG. 144.

Example. If the gear having 48 teeth, Fig. 144, makes 80 R.P.M., what will be the number of revolutions per minute of the small gear, which has 24 teeth?

Solution.

$$\begin{array}{cccccccc} \text{(teeth)} & & \text{(teeth)} & & \text{(Rev.)} & & \text{(Rev.)} \\ 48 & : & 24 & = & x & : & 80 \\ (1) & & (2) & & (4) & & (3) \end{array}$$

$$24x = 3840$$

$x = 160$ revolutions per minute. Ans.

113. Gear Trains. To distinguish gears in a train, the one which supplies power is called the driver, while

the other is the follower or driven gear. Proportion gives us a simple way of finding the effect of several gears in a train. When there are only two gears in the train, the proportion is *inverse* between the number of teeth and revolutions, and may be stated in general as follows:

Teeth on driver : *Teeth on driven* = *Rev. of driven* : *Rev. of driver.*

The effect of several gears in a train may be made clear by the following example:

Example. In the gear train shown in Fig. 145, suppose the first driver of 96 teeth makes 30 R.P.M. What will be the number of R.P.M. of the last gear in the train, which has 44 teeth?

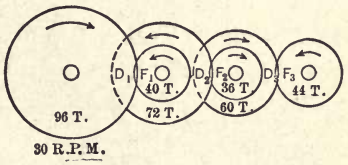


FIG. 145.—Gear Train.

In figure, D_1, D_2, D_3, are the *driving gears*, and F_1, F_2 and F_3 are the *driven gears* or followers. By a mathematical combination of the different proportions between the gears, which it is not necessary to trace out, we may obtain a final proportion which will include all the gears in the train.

Product of teeth on all drivers : *Product of teeth on all driven gears* = *Rev. per minute of last driven* : *Rev. per minute of first driver.*

This proportion may be more conveniently written in fractional form:

$$\frac{\text{Product of teeth on all drivers}}{\text{Product of teeth on all driven}} = \frac{\text{R.P.M. of last driven}}{\text{R.P.M. of first driver}}.$$

Solution.

$$\frac{\cancel{24}^{\,2} \times \cancel{72}^{\,3} \times \cancel{60}}{\cancel{40} \times \cancel{36} \times \cancel{44}_{11}} = \frac{x}{30}$$

$$\frac{72}{11} = \frac{x}{30}.$$

If

$$72 : 11 = x : 30$$

$$11x = 72 \times 30$$

$$11x = 2160$$

$$x = 196\tfrac{4}{11} \text{ R.P.M.} \qquad \text{Ans.}$$

114. Pulley Trains. When several pulleys are connected by belting so as to run by power from one of the

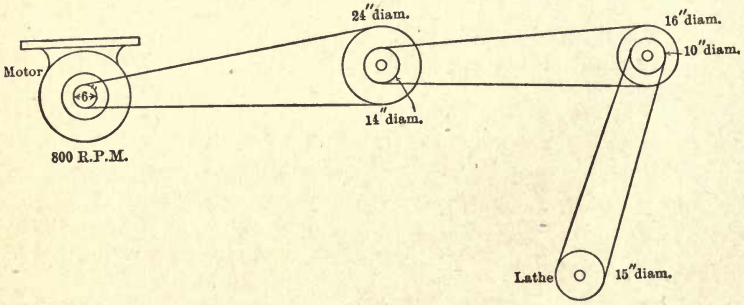


Fig. 146.—Arrangement of Belts for a Lathe.

pulleys, the arrangement is known as a pulley train. The problem of a pulley train is exactly the same as for a gear train, except that in the case of pulleys we use the diameters of the pulleys instead of the number of teeth on the gears.

Example. Find the number of revolutions per minute

of the lathe shown in Fig. 146, when driven by the electric motor and pulley train.

Solution.

$$\frac{\text{Product of diam. of all drivers}}{\text{Product of diam. of all driven}} = \frac{\text{R.P.M. of last driven}}{\text{R.P.M. of first driver}}.$$

$$\frac{\overset{7}{\cancel{6}}\times\overset{5}{\cancel{14}}\times\cancel{10}}{\underset{4}{\cancel{24}}\times\underset{4}{\cancel{16}}\times\underset{3}{\cancel{15}}} = \frac{x}{800}$$

$$\frac{7}{48} = \frac{x}{800}$$

$$48x = 5600$$

$$x = 116.+\text{R.P.M.} \qquad \text{Ans.}$$

PROBLEMS

1. If a workman taps 25 nuts in 45 minutes, how long will it take him to tap 100 nuts at the same rate?

2. A pulley 24 ins. in diameter is running at 210 R.P.M. and is connected by a belt to a pulley 14 ins. in diameter. How many revolutions per minute will the smaller pulley make?

3. A line shaft runs at 175 R.P.M. A 28-in. pulley on the line shaft is belted to a countershaft which makes 200 R.P.M. Find the size of the pulley on the countershaft.

4. If 250 castings cost \$75, what will 150 castings of the same kind cost?

5. A gear having 96 teeth is meshed with a gear having 40 teeth. If the large gear makes 65 R.P.M., how many will the small one make?

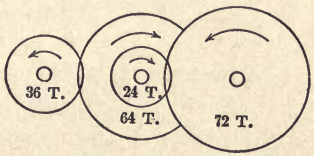


FIG. 147.

6. Find the number of revolutions per minute of the last gear in the train in Fig. 147, when the gear having 72 teeth makes 81 R.P.M.

7. How many revolutions per minute will the lathe in Fig. 148 make?

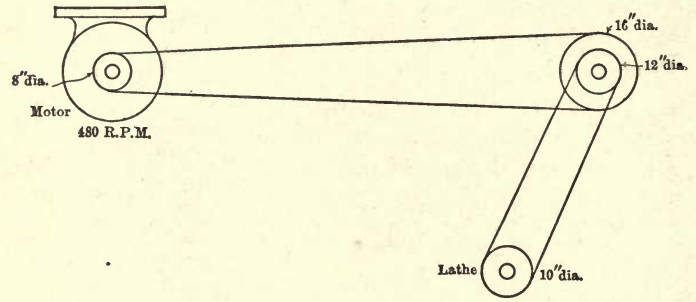


FIG. 148.

8. Find the number of revolutions per minute of the lathe spindle in Fig. 149, when the cone pulley runs at the rate of 140 R.P.M.

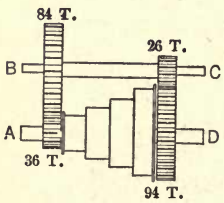


FIG. 149.—Back Gears on Lathe.

115. Belting. A common method of transmitting power from one place to another in the shop is by means of leather belting passing over pulleys of different sizes. In modern shops this method is being rapidly displaced by attaching electric motors directly to each machine.

1. *Approximate Length of Open Belt.* If the pulleys and shafting are already in position, the easiest way to find the length of an open belt is to pass a tape line around the pulleys in the position that the belt is to occupy. The sag of the tape line will be about the same as that of the belt.

It is sometimes necessary to calculate the length of an

open belt, generally when the two pulleys are of unequal size, as in Fig. 150.

The straight part of the belt is evidently a little more than *twice* the distance l. The part of the belt on the large pulley covers a little more than half the circumference,

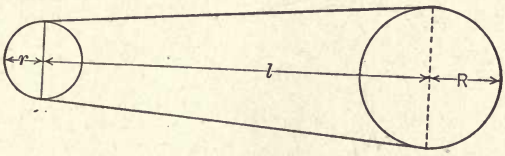


FIG. 150.—Open Belt.

and on the small pulley a little less than half the circumference. This varies somewhat with the tightness of the belt. If we take about $3\frac{1}{4}$ times the sum of the radii of the pulleys and add twice the distance between their centers to this, we shall have the approximate length of the belt.

$$\textit{Length of open belt} = 3\tfrac{1}{4}(R+r)+2l.$$

Example. Two pulleys 24 and 16 ins. respectively in diameter, have their centers 18 ft. apart. Find the length of the open belt necessary to connect them.

Solution. $3\frac{1}{4}(12+8)+2(18\times12)$

$=3\frac{1}{4}(20)+2(216)$

$=65+432=497$ ins. or 41.41 ft. Ans.

Pulleys are placed 20 to 25 ft. apart for ordinary belts and somewhat nearer for small belts.

2. *Approximate Length of a Crossed Belt.* The length of a crossed belt, Fig. 151, is a little more than that of an open belt. If we multiply the sum of the radii of the

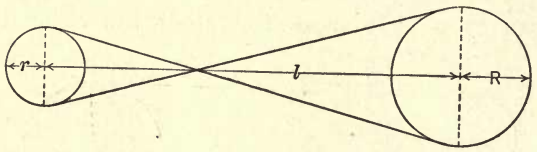


FIG. 151.—Crossed Belt.

pulleys by $3\frac{3}{8}$ instead of $3\frac{1}{4}$ we shall have the approximate length of a crossed belt.

$$\textit{Length of crossed belt} = 3\tfrac{3}{8}(R+r)+2l.$$

Example. Find the length of a crossed belt to connect a 24- and a 16-in. pulley with centers 18 ft. apart.

Solution. $3\frac{3}{8}(12+8)+2(18\times12)$

$=3\frac{3}{8}(20)+2(216)$

$=67.5+432=499.5$ in. or 41.62 ft. Ans.

116. A. Thickness of Belting. Single belting is generally used on pulleys under 12 ins. in diameter and double belting on pulleys over 12 ins. in diameter. Thicker belting is used on pulleys 25 ins. or more in diameter.

B. Speed of Belting. The speed of a belt is found by multiplying the circumference of either pulley by its number of revolutions per minute. For ordinary use, belts on machines run from 1000 to 2000 ft. per minute, while main driving belts run at about double this speed. A belt

should not be run over 6000 ft. per minute. In calculating the actual speed of a belt, about 2% is allowed for slipping on the pulleys.

Example. Find the speed of a belt running over a pulley 18 ins. in diameter which is turning at the rate of 326 R.P.M.

Solution.

Circumference of pulley $= 18 \times 3\frac{1}{7} = 56.57$ ins. $= 4.7$ ft.

Speed $= 4.7 \times 326 = 1532+$ ft. per minute.

Less $2\% = 1501$ ft. per minute. Ans.

C. Horse-power of Belting. The power transmitted by a belt depends on the *pull* of the belt and the *speed* with which it travels. A pull of about 35 lbs. per inch of width is allowable for a single belt and 70 lbs. per inch of width for a double belt. To find the total power transmitted by a belt it is necessary to calculate the foot-pounds of work done in one minute. This result divided by 33,000 will give the horse-power.

Example. Find the horse-power transmitted by a single belt 4 ins. wide, running over a pulley 10 ins. in diameter which is turning 400 R.P.M.

Solution.

Force or pull $= 35 \times 4 = 140$ lbs.

Circumference of pulley $= 10 \times 3\frac{1}{7} = 31.43$ ins. $= 2.62$ ft.

Speed $= 2.62 \times 400 = 1048$ ft. per minute.

Power = force × speed $= 140 \times 1048 = 146{,}720$ ft.-lbs. per min.

Horse-power $= 146{,}720 \div 33{,}000 = 4.44$. Ans.

PROBLEMS

1. Find the length of an open belt to connect two pulleys 24 and 14 ins. in diameter with their centers 22 ft. apart.

2. Find the length of a crossed belt for connecting two pulleys 20 and 12 ins. in diameter with their centers 16 ft. apart.

3. A pulley 20 ins. in diameter makes 240 revolutions per minute. Find the speed of a belt running over this pulley allowing 2% for the slipping of the belt.

4. A single belt 5 ins. wide is running at a speed of 1800 ft. per minute. Find the horse-power transmitted.

5. How wide should a single belt be to transmit 10 horse-power when running at a speed of 1570 ft. per minute?

6. An 8-inch double belt is running over a 22-in. pulley turning at the rate of 210 R.P.M. Find the horse-power transmitted.

7. A single belt 4 ins. wide transmits 5 horse-power. Find the speed at which it is running.

REVIEW PROBLEMS

1. Two gears have a speed ratio of 1 to 4. If the larger gear has 96 teeth, how many teeth has the smaller gear?

2. Find the ratio between the R.P.M. of the cone pulley and the spindle of the lathe shown in Fig. 149, when the back gears are in mesh, if gear A has 31 teeth, gear B 85 teeth, gear C 32 teeth, gear D 84 teeth, and the cone pulley is running at 185 R.P.M.

3. If a gear having 40 teeth drives one having 100 teeth, what is the ratio of speeds of the gears?

4. A motor lifts an elevator weighing $1\frac{1}{2}$ tons (2000 lbs. each) to the top of a building 300 ft. high in 2 minutes. What horse-power is exerted by the motor?

5. Find the approximate length of an open belt to connect two pulleys 16 and 28 ins. respectively, with centers $17\frac{1}{2}$ ft. apart.

6. A driving pulley 32 ins. in diameter on a line shaft running 180 R.P.M. is belted to a 14-in. pulley on a countershaft. The countershaft has a pulley 16 ins. in diameter belted to a 5-in. pulley on an emery wheel. How many R.P.M. will the emery wheel make?

7. How wide a double belt will transmit 15.4 horse-power over a pulley 22 ins. in diameter when running at 180 R.P.M.?

8. Find the approximate length of a crossed belt connecting two pulleys 32 and 20 ins. in diameter with a distance of $14\frac{1}{2}$ ft. between centers.

9. What horse-power is transmitted by an 8-in. double belt running over a 28-in. pulley at 120 R.P.M.?

10. A double belt 10 ins. wide transmits 20 horse-power. What is its speed in feet per minute?

11. How much work is done when an engine lifts 10 tons of coal (2000 lbs. each) from a mine 660 feet deep?

12. Find the horse-power of a steam engine with cylinder 22 ins. in diameter and 48-in. stroke, running at 60 R.P.M., with an average steam pressure of 60 lbs. per square inch on the piston.

13. Find the brake horse-power of an engine exerting a pressure of 80 lbs. on a pair of scales at a radius of 5 ft. when running at 450 R.P.M.

14. How long will it take a 12 horse-power engine to lift 4090 lbs. 200 ft. high?

15. How many revolutions per minute will a steam engine make, which is developing 259 horse-power, and has a cylinder 18 ins. in diameter, with 36-in. stroke, if the average pressure on the piston is 80 lbs. per square inch?

16. If an engine is developing 18 horse-power, how many foot-pounds of work can it do in 30 minutes?

17. Find the speed ratio between A and C in Fig. 152, when A makes 175 R.P.M.

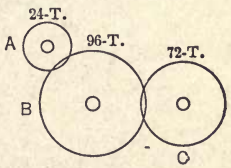


FIG. 152.

18. How many R.P.M. should a countershaft with a 5-in. pulley make in order that a grindstone having a pulley 20 ins. in diameter shall turn at the rate of 48 R.P.M.?

19. If 40 castings are bought for \$30, how many of the same kind may be bought for \$69?

20. How wide a single belt will transmit 1.7 horse-power when running over a pulley 11 ins. in diameter turning at 185 R.P.M.?

CHAPTER XIV

GEAR CALCULATIONS

117. Spur Gears. A **gear** is a wheel with teeth on its circumference.

A spur gear has the teeth on its circumference projecting radially, that is, they point from the center like the spokes of a wheel. This is the most common form of gear. A small spur gear is often called a **pinion.**

If the teeth of a gear should be made very small, the

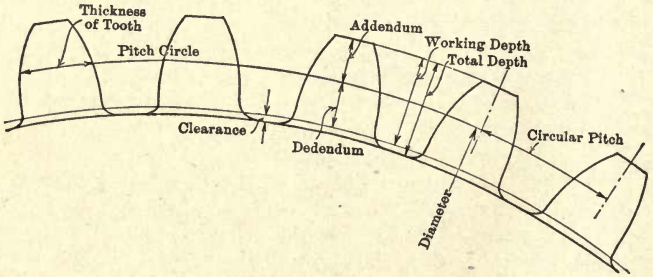


FIG. 153.—Terms Applying to a Gear.

tops and bottoms of the teeth would gradually approach a line about half way between them. This line is know as the **pitch line** or **pitch circle,** Fig. 153. Gear calculations are usually made with reference to the pitch circle.

The **diameter** of a gear always means the diameter from one side of the pitch circle to the other side, unless otherwise stated. It is sometimes called the **pitch diameter.**

Diametral pitch is the number of teeth to every inch of diameter of the gear. If a gear has 48 teeth and is 4 ins. in diameter, it is a $\frac{48}{4} = 12$-*pitch* gear, since there are 12 teeth

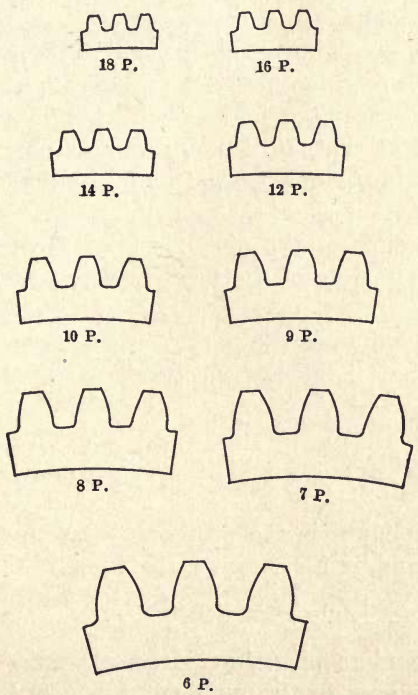


FIG. 154.—Relative Size of Gear Teeth.

to every inch of diameter. If a gear has 42 teeth and is 3 ins. in diameter it is a $\frac{42}{3} = 14$-*pitch* gear.

In order that gears of different diameters may run together or *mesh*, their teeth must all be of the same size. By having the same diametral pitch in each gear of a set,

the teeth are all of the same size, and the *number of teeth* varies directly with the diameter of the gear. For example in a 12-pitch set of gears, the gear 2 ins. in diameter will have $12 \times 2 = 24$ teeth, the gear 3 ins. in diameter will have $12 \times 3 = 36$ teeth, and so on.

Fig. 154 gives the comparative size of gear teeth for some of the common pitches. A large wheel may have a small diametral pitch, and a small wheel may have a large diametral pitch, so that diametral pitch in a way represents the size of the teeth. *Do not confuse diametral pitch with pitch diameter.*

Circular pitch is the distance from the middle of one tooth to the middle of the next, measured along the *pitch circle.*

The **addendum** is the part of a gear tooth outside the pitch circle. It is always a certain part of the *size* of the tooth, and is best stated in inches as $\frac{1}{\text{pitch}}$. In a 10-pitch gear the addendum would be $\frac{1}{10}$ in., in a 14-pitch gear $\frac{1}{14}$ in., etc.

The **dedendum** is the part of a gear tooth below the pitch circle and, in the same gear, is equal to the addendum. The dedendum is also stated as $\frac{1}{\text{pitch}}$.

The addendum plus the dedendum makes the **working depth** of the gear tooth, but not the whole depth. **Clearance** is provided at the bottom of the space between the gear teeth. Clearance is usually taken in the machine shop as $\frac{1}{10}$ of the width of the tooth on the pitch circle. The addendum plus the dedendum plus the clearance gives the **total depth** of the tooth.

The **thickness** of a tooth is designed to be a trifle less than the space between the teeth in order to give a small

side clearance or backlash, but in practice the thickness of the tooth and the space are made the same. The thickness of the tooth and the space would therefore each be $\frac{1}{2}$ the circular pitch.

The **root diameter** is the diameter measured from the bottom of a space on one side of a gear to the bottom of the space on the opposite side.

The **outside diameter** is the diameter measured at the extreme outside of the gear.

118. To Find Circular Pitch from Diametral Pitch. Example. A gear has a diametral pitch of 12. What is the circular pitch?

Solution. In any circle the circumference is 3.1416 times the diameter, therefore for every inch in the diameter there will be 3.1416 ins. on the circumference. The diametral pitch is 12, so 3.1416 ins. on the circumference will be occupied by 12 teeth. The circular pitch is the distance that *one* of these teeth will occupy (that is, a tooth and a space), therefore:

$$3.1416 \div 12 = .2618\text{-in. circular pitch.} \qquad \text{Ans.}$$

$$\text{Circular pitch} = \frac{3.1416}{\text{diametral pitch}}.$$

119. To Find Diametral Pitch from Circular Pitch. Example. The circular pitch of a gear is .3927 in. What is the diametral pitch?

Solution. For every inch in the diameter there will be a certain number of teeth on the circumference. This number of teeth on the circumference will occupy 3.1416 ins. Since a tooth and a space occupy .3927 in., there will be as many teeth as .3927 is contained in 3.1416 or $3.1416 \div .3927 = 8$ teeth. This is the number of teeth

for every inch of diameter and is therefore the diametral pitch.

$$\text{Diametral pitch} = \frac{3.1416}{\text{circular pitch}}.$$

PROBLEMS

1. The circular pitch of a gear is .2244 in. What is the diametral pitch?
2. The diametral pitch of a gear is 16. Find the circular pitch.
3. What number would be obtained in every case if the circular pitch were multiplied by the diametral pitch?
4. The circular pitch of a gear is .3491 in. What is the diametral pitch?
5. The diametral pitch of a gear is 11. Find the circular pitch.

120. To Find the Thickness of a Gear Tooth. Example. What is the thickness of a tooth at the pitch circle, of a 14-pitch gear 4 ins. in diameter?

Solution. Number of teeth $= 14 \times 4 = 56$.
Circular pitch $= 3.1416 \div 14 = .2244$.
Thickness of tooth = circular pitch $\div 2$.
$.2244 \div 2 = .1122$ in. Ans.

$$\text{Thickness of tooth} = \frac{\text{Circular pitch}}{2}.$$

Short Solution. The circular pitch of a 1-pitch gear is $3.1416 \div 1 = 3.1416$ in. The thickness of a tooth on a 1-pitch gear is $\frac{1}{2}$ of this: $3.1416 \div 2 = 1.57$. The thickness of tooth on a 2-pitch gear would be $\frac{1.57}{2}$, on a 3-pitch gear $\frac{1.57}{3}$ and so on. We may therefore use the number 1.57

as a *constant*, and get the thickness of tooth on any pitch gear by dividing this constant by the pitch of the gear. In the above example:

$$\frac{1.57}{14} \div .1121 \text{ in.} + \text{Circular pitch.} \qquad \text{Ans.}$$

121. To Find the Total Depth of a Tooth. Example. What is the total depth of a tooth on a 14-pitch gear?

Solution. $\text{Addendum} = \frac{1}{P} = \frac{1}{14}$ in.

$$\text{Dedendum} = \frac{1}{P} = \frac{1}{14} \text{ in.}$$

Thickness of tooth $= \frac{1}{2}$ of circular pitch.

$$\frac{1}{2} \times \frac{3.1416}{14} = \frac{3.1416}{28} = .1122 \text{ in.}$$

$$\text{Clearance} = \frac{1}{10} \text{ of thickness}$$

$$\frac{1}{10} \times .1122 = .01122 \text{ in.}$$

$$\text{Total depth} = \text{addendum} + \text{dedendum} + \text{clearance} = \frac{1}{14}$$

$$+ \frac{1}{14} + .01122 = .0714 + .0714 + .01122 = .154 \text{ in.} \qquad \text{Ans.}$$

Short Solution. In any gear the addendum is $\frac{1}{P}$, the dedendum $\frac{1}{P}$, and the clearance $\frac{1}{10}$ of the thickness of tooth.

Therefore total depth $= \frac{1}{P} + \frac{1}{P} + \frac{.157}{P} = \frac{2.157}{P}$.

The number 2.157 may be used as a *constant* to find the depth of tooth on any pitch gear. When divided by the pitch of the gear it will give the total depth of the tooth. In the above example:

$$\frac{2.157}{14} = .154 \text{ in.} \quad \text{Total depth of tooth.} \qquad \text{Ans.}$$

PROBLEMS

1. Find the thickness of tooth on a 12-pitch gear.
2. Calculate the thickness of tooth on a gear of 18 pitch.
3. Does the number of teeth on a gear with the same pitch make any difference in the thickness of the tooth?
4. Find the total depth of a tooth on a 10-pitch gear.
5. Find the total depth of tooth on an 8-pitch gear.
6. Find the total depth of tooth on a 12-pitch gear.

122. To Find the Outside Diameter of a Gear Blank. Example. What should be the outside diameter of a gear blank for 56 teeth, 14 pitch?

Solution.

Diameter of pitch circle $= \frac{56}{14} = 4$ ins.

Addendum $= \frac{1}{14}$ in. for one side, or $\frac{2}{14}$ in. for both sides.

Total outside diameter $= 4 + \frac{2}{14} = 4.142$ ins. Ans.

Short Solution. Notice in the above example that the number of teeth (56) is divided by the pitch (14), also the addendum on both sides is $\frac{2}{14}$. We may combine this division as follows:

$$\frac{56}{14} + \frac{2}{14} = \frac{56+2}{14} = \frac{58}{14} = 4.142 \text{ ins.} \qquad \text{Ans.}$$

Since the addendum on two sides is always $\frac{2}{\text{pitch}}$ we may find the outside diameter of any gear as follows:

$$\text{Outside diameter} = \frac{\text{Number of teeth} + 2}{\text{pitch}}.$$

PROBLEMS

1. Find the outside diameter of a gear blank for 96 teeth, 14 pitch.
2. What is the outside diameter of a gear blank for 60 teeth, with a circular pitch of .2618 in.?
3. What should be the outside diameter of a gear blank for 72 teeth, 12 pitch?
4. Find the outside diameter of a gear blank for 128 teeth, 8 pitch.
5. Find the outside diameter of a gear blank for 100 teeth, with a circular pitch of .1963 in.

123. To Find Number of Teeth when Outside Diameter and Pitch are Given. Example. What is the number of teeth on a gear 7 ins. outside diameter, 14 pitch?

Solution. Twice the addendum $= \frac{2}{14}$ in.
Diameter to pitch circle $= 7 - \frac{2}{14} = 6\frac{6}{7}$ ins.
Number of teeth $= 14 \times 6\frac{6}{7} = 96$ teeth. Ans.

124. To Find Pitch when Outside Diameter and Number of Teeth are Given. Example. What is the pitch of a gear having 72 teeth and measuring 6.163 ins. outside diameter?

Solution.

$$\text{Outside diameter} = \frac{\text{Number of teeth} + 2}{\text{Pitch}}.$$

Substituting known values in this expression we have:

$$6.163 = \frac{72+2}{\text{Pitch}}.$$

Let x stand for the pitch, then:

$$6.163 = \frac{74}{x}.$$

Multiplying both sides by x

$$6.163x = \frac{74x}{x},$$

$$x = \frac{74}{6.163} = 12 \text{ pitch.} \qquad \text{Ans.}$$

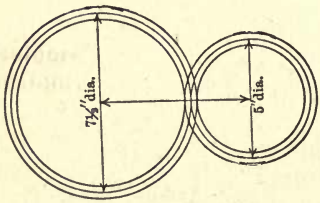


FIG. 155.—Distance Between a Pair of Gears.

125. To Find the Distance between Centers of a Pair of Gears. Example. What is the distance between centers of a pair of gears having 60 teeth and 40 teeth respectively, 8 pitch, as shown in Fig. 155.

First Solution.

Diameter of 40-tooth gear $= 40 \div 8 = 5$ ins. (to pitch circle);
Diameter of 60-tooth gear $= 60 \div 8 = 7\frac{1}{2}$ ins.;
Distance between centers $= \frac{1}{2}$ diameter of large gear plus $\frac{1}{2}$ diameter of the small gear.

$$2\tfrac{1}{2} + 3\tfrac{3}{4} = 6\tfrac{1}{4} \text{ ins. distance.} \qquad \text{Ans.}$$

Second Solution. If 60 teeth represent the *diameter* of the large gear and 40 teeth the diameter of the small gear, the *radius* of each gear will be represented by $\frac{1}{2}$ that number of teeth on each.

$$\frac{60+40}{2}=50 \text{ teeth.}$$

The relation of the radius of each gear to this number of teeth is the same as that of the diametral pitch to the whole number of teeth. The distance between centers, which is the sum of the two radii, will then be

$$\frac{50}{8}=6\tfrac{1}{4} \text{ ins.} \qquad \text{Ans.}$$

This may be expressed:

$$\text{Distance between centers}=\frac{\text{Sum of teeth on both gears}}{2\times\text{pitch}}.$$

126. Rack and Internal Gears. A **rack** may be defined as a gear whose pitch circle has become a straight line. Racks and internal-gear teeth are made on the same principles as spur-gear teeth. For further information regarding them, consult any standard book on gearing.

127. Bevel Gears. These are gears whose teeth mesh on a pitch *cone* instead of a pitch cylinder.

Mitre gears are bevel gears of the same size, having the same number of teeth, and their shafts at right angles to each other. The same relations hold between diameter, pitch, number of teeth, etc., in bevel gears as in spur gears, and calculations are made in the same way. In all calculations the diameter of the bevel gear is taken at the bottom of the pitch cone. For the angles of these

gears, laying out, etc., consult some standard book on gearing.

128. Worm Gears and Wheels. A **worm** is a screw cut in such a manner that it will mesh with a wheel called a **worm wheel.** The rate of turning of the worm wheel depends upon the distance that the thread of the worm advances for each revolution of the worm.

Calculations on worms and worm wheels are too complicated to be treated in this book.

REVIEW PROBLEMS

1. How many teeth should there be on a 16-pitch gear $3\frac{3}{4}$ ins. in diameter?
2. If the diametral pitch of a gear is $2\frac{1}{2}$, what is the circular pitch?
3. If the circular pitch on a gear is .2094, what is the diametral pitch?
4. What is the diametral pitch of a gear measuring 2.857 ins. to the pitch circle and having 40 teeth?
5. Find the outside diameter of a gear blank for 69 teeth, 14 pitch.
6. What is the outside diameter of a gear blank with 80 teeth and a circular pitch of .2618 in.?
7. A gear measures $9\frac{1}{3}$ ins. outside diameter and has 82 teeth. What is the pitch of the gear?
8. Calculate the total depth of tooth of a gear of 6 pitch.
9. What is the total depth of tooth of a gear having a circular pitch of .4488 in. and 82 teeth?
10. A gear measures $17\frac{7}{8}$ ins. outside diameter and has 85 teeth. Find the pitch of the gear.
11. The circular pitch of a gear is .7854. What is the thickness of a tooth?
12. The circular pitch of a gear is .2417. What is the addendum?
13. The outside diameter of a gear is 8 ins. and it is 12 pitch. Find the diameter at the bottom of the teeth, allowing for clearance.

14. If a gear is 16 pitch, what is the working depth of the tooth?

15. Two gears in mesh have 56 teeth and 72 teeth respectively. If they are 12 pitch, what is the distance between their centers?

16. A gear having 180 teeth meshes with one having 45 teeth. If the large gear makes one revolution, how many revolutions will the small gear make?

17. What is the outside circumference of a 12-pitch gear having 70 teeth?

18. Two gears in mesh have 44 teeth and 96 teeth respectively. If they are 14-pitch, what is the distance between their centers?

19. A 10-pitch gear measuring 9 ins. outside diameter will have what number of teeth?

20. What is the outside diameter of a gear blank for 66 teeth, 12 pitch?

CHAPTER XV

MILLING MACHINE

129. Cutting Speed. The cutting speed for work on a milling machine depends principally on the size and number of revolutions per minute of the milling cutter. The diameter of the cutter corresponds to the diameter of a piece of work in a lathe and the cutting speed is reckoned in the same way, the only difference being that on the lathe it is the piece of work that is revolving, while on the milling machine the work is stationary and the cutter is revolving.

Example. A milling cutter 4 ins. in diameter makes 50 R.P.M. Find the cutting speed.

Solution.

$4 \times 3.1416 = 12.57$ ins. circum. of cutter.

$12.57 \times 50 = 628.5$ ins. or $52.3+$ ft. cutting speed. Ans.

130. Feed. The feed for work on a milling machine depends mainly on the clearance and pitch of teeth on the cutter. A shallow, roughing cut may be taken more rapidly than a deep cut. In general the rule for feed is the same as for a lathe; roughing cuts should be taken with a coarse-pitch cutter at slow speed and with heavy feed, while finishing cuts should be taken with a fine-pitch cutter at fast speed and with light feed.

The feed is easily calculated by knowing the lead of the feed screw on the table or platen of the machine.

Example. If the lead of the screw on the table of a

milling machine is $\frac{1}{4}$ in. and it is turned at the rate of 5 R.P.M., what is the feed?

Solution. $\frac{1}{4} \times 5 = \frac{5}{4} = 1\frac{1}{4}$ inches per min. Ans.

131. Table of Speeds and Feeds. Mr. W. J. Kaup gives the following speeds and feeds in his book on "Machine Shop Practice":

	Speed in ft. per min.	Feed in ins. per min.
Tool steel (annealed)	28	$\frac{6}{10}$
Soft steel	42	$\frac{8}{10}$
Cast iron	50 to 60	$1\frac{1}{4}$
Brass	90 to 100	2
Bronze	78	$1\frac{1}{2}$

PROBLEMS

1. A milling cutter 2 ins. in diameter is running at 80 R.P.M. Find the cutting speed.

2. A milling cutter $3\frac{3}{4}$ ins. in diameter is running at 70 R.P.M. Find the cutting speed.

3. A milling cutter turning at the rate of 25 R.P.M. is cutting at a speed of 36 ft. per minute. Find the diameter of the cutter.

4. A milling cutter 4 ins. in diameter is cutting at a speed of 76.69 ft. per minute. Find its number of revolutions per minute.

5. The screw on the table of a milling machine has a lead of $\frac{1}{4}$ in. How many R.P.M. must it make to give a feed of 1 in. per minute?

6. If the dial at the end of the lead screw in problem 5 is to be graduated in thousandths of an inch, how many divisions will there be on the circumference?

7. Suppose a lead screw has a double thread with a pitch of $\frac{1}{4}$ in. What feed in inches per minute will there be when it is turning at 3 R.P.M.?

132. Indexing. Indexing is the process of dividing the circumference of a piece of circular work on a milling machine into equal parts. It is accomplished by means of

an **index plate** and an **index crank** on the dividing head of the machine. Of several forms of indexing, only three are important: *direct indexing, simple indexing,* and *differential indexing.*

133. Direct Indexing. This is done by means of an index plate fastened directly to the spindle in the dividing head. When this method is used, the worm is thrown out of mesh with the worm wheel. The index plate used for direct indexing usually has 24 holes and is held in position by a stop pin in the frame of the machine. The plate

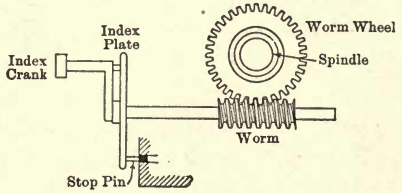


FIG. 156.—Index Plate and Mechanism.

is rotated by hand to any desired position by withdrawing the stop pin.

Any number of divisions which will divide evenly into 24 may be obtained with this plate, such as two sides, square, three sides, hexagon and octagon.

134. Simple Indexing. In simple indexing, the spindle is turned by means of a worm and worm wheel as shown in Fig. 156. The index plate is kept from turning by the stop pin back of it. The common ratio between the index crank and spindle is 40 to 1, that is, it takes 40 complete turns of the index crank for one complete turn of the spindle. Some milling machines have a ratio of 60 to 1.

In order to obtain different divisions, the index plate has circles of holes bored in it, with a different number of

holes in each circle. The following plates will allow a wide range of divisions, taking *all* numbers up to 58, *even* numbers up to 100, and many others as high as 360.

First plate:	34, 39, 46, 51, 58.
Second plate:	36, 41, 47, 53, 60.
Third plate:	37, 42, 48, 54, 62.
Fourth plate:	38, 43, 49, 56, 66.

The use of the plates may be shown in the following way:

Example. Find the plate to be used and the spacing required to cut 36 teeth on a gear blank, when the dividing head has a ratio of 40 to 1.

Solution. 40 turns of index crank = 1 rev. of spindle.

For 36 teeth, these 40 turns must be divided into 36 parts.

$$40 \div 36 = 1\tfrac{1}{9} \text{ turns for each tooth.}$$

The $\frac{1}{9}$ turn may be taken on any circle of holes into which 9 will go without a remainder. Take the third plate in the list given and the circle of 54 holes. Then $54 \div 9 = 6$ holes.

The 36 divisions will therefore be obtained by making one complete turn of the index crank and going 6 holes on the 54 circle for each division.

Example. Find the plate to be used and the spacing required for 96 divisions when the ratio of the dividing head is 40 to 1.

Solution. 40 turns must be divided into 96 parts.
Each division will be $40 \div 96 = \frac{5}{12}$ of a turn.
Use the third plate and circle of 48 holes.
$\frac{5}{12} \times 48 = 20$ holes.

For each division move 20 holes on the circle of 48 holes.

135. The Dividing Sector. In order to avoid mistakes in counting holes each time, a sector is provided having two arms, which can be adjusted to any angle of opening, Fig. 157. The holes are counted between the arms. Several things must be observed in using this sector.

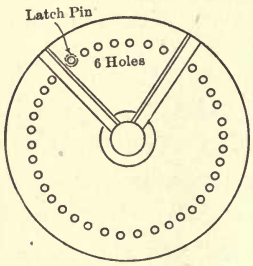


FIG. 157.—Dividing Sector for Index Plate.

Caution 1. Never count the hole in which the latch pin of the index crank is placed. For 6 holes there should actually be 7 holes between the arms of the sector, but one is filled by the latch pin.

Caution 2. Since there is likely to be some backlash in the moving parts of the dividing head, be careful never to go *beyond* a hole with the latch pin. If a mistake is made, go back at least half a turn of the index crank and then come back to the proper hole.

Caution 3. Always turn the sector forward to its new position immediately after inserting the latch pin, so as to avoid forgetting to do so after the cut is taken.

PROBLEMS

1. How would you index, by the direct method, for milling a square on the end of a tap, when the index plate has 24 holes?

2. Calculate the turns and holes necessary for milling three equally spaced sides on a round bar, using simple indexing, with a ratio of 40 to 1 in the dividing head.

3. How would you index for milling a hexagon by the direct method, if the index plate has 24 holes?

4. Find the complete turns and the circle and number of holes for indexing 14 divisions, when the ratio of the dividing head is 60 to 1.

5. What indexing would be used for 105 divisions when the ratio of the dividing head is 40 to 1?

6. What indexing would be used for 120 divisions, when the ratio of the dividing head is 60 to 1?

7. What indexing would be used for 180 divisions, when the ratio of the dividing head is 40 to 1?

136. Differential Indexing. It sometimes happens that a number of divisions is required that cannot be obtained with the ordinary index plates. In such cases, a process of differential indexing may be used, where the spindle and index plate are connected together by change gears, so that

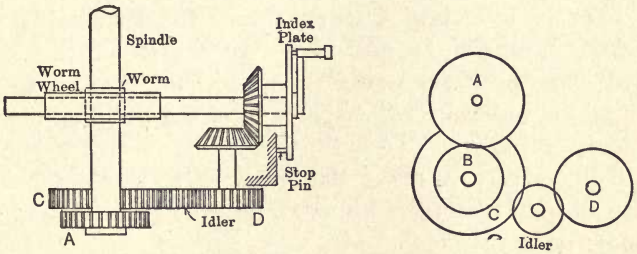


FIG. 158.—Mechanism for Differential Indexing.

the index plate can be moved by means of the worm and spindle.

The index plate is driven by a train of gears from the end of the spindle as shown in Fig. 158. The stop pin is first removed from the index plate to allow it to turn freely; it may then be made to revolve by the change gears, either in the same direction as the index crank or in the opposite direction.

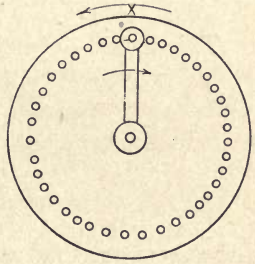


FIG. 159.

If the index plate is stationary, the index crank will pass the point X 40 times, Fig. 159, for one complete turn of the spindle. Suppose that the change gears cause the plate to turn once in

the same direction as the crank for one turn of the spindle. In that case the crank will pass point X only 39 times for one turn of the spindle, because the one turn of the plate will make up for the fortieth turn. In the same way, if the plate turns in the opposite direction to the crank, it will be necessary for the crank to pass point X 41 times for one turn of the spindle, in order to make up for the plate turning once in the opposite direction.

137. To Calculate Change Gears for Differential Indexing. Example 1. Calculate the moves of the index crank and the change gears necessary for 99 divisions by differential indexing, when the ratio of the dividing head is 40 to 1.

Solution. If we make 100 moves of 16 holes each on a plate with 40 holes, the worm would turn 40 times and the spindle once.

$$\frac{16}{40}\times 100=\frac{1600}{40}=40 \text{ rev. of worm.}$$

If 99 moves are made:

$$\frac{16}{40}\times 99=39.6 \text{ rev. of worm.}$$

This lacks .4 of being 40.

We may obtain this .4 of a turn by moving the plate in the *same direction* as the crank by means of suitable gears in the train.

$$.4 \text{ is the same ratio as } \frac{1}{2.5}.$$

Since there are four gears in the train we split this ratio:

$$\frac{1}{2.5}=\frac{2}{2.5}\times\frac{1}{2}.$$

Multiply the parts of this ratio by any numbers that will give the gears in stock.

$$\frac{2}{2.5}\times\frac{16}{16}=\frac{32}{40}$$ gear A (driver)
gear B (driven)

$$\frac{1}{2}\times\frac{28}{28}=\frac{28}{56}$$ gear C (driver)
gear D (driven). (See Fig. 158.)

With this gearing, 99 moves of 16 holes each on a circle of 40 holes will give the required divisions. (8 holes on a 20 circle will give the same result.)

Example 2. Calculate the differential indexing for 107 divisions on an index head with a ratio of 40 to 1.

Solution. Try 16 holes on the 40 circle as in example 1.

Then $\frac{16}{40}\times 107=42.8$ rev. of the worm.

Since this is 2.8 too many revolutions, the plate must be turned in the *opposite direction.* This is done by putting an *idler* in the gear train, which does not affect the ratio. The ratio is 2.8 to 1.

Split the ratio: $\frac{2.8}{1}=\frac{2.8}{2}\times\frac{2}{1}$,

$$\frac{2}{1}\times\frac{32}{32}=\frac{64}{32}$$ gear A (driver)
gear B (driven)

$$\frac{2.8}{2}\times\frac{20}{20}=\frac{56}{40}$$ gear C (driver)
gear D (driven).
(See Fig. 158.)

Therefore 107 moves of 16 holes each on a 40-hole circle

will give the required divisions, or 8 holes on a 20-hole circle.

Example 3. Calculate the differential indexing for 139 divisions on an index head with a ratio of 40 to 1.

Solution. Try until a combination is found to fit the case. 140 moves of 12 holes each on a 42-hole plate will give:

$$\frac{12}{42}\times 140 = 40 \text{ rev. of worm.}$$

For 139 moves

$$\frac{12}{42}\times 139 = 39\tfrac{15}{21} \text{ rev. of spindle.}$$

This lacks $\frac{6}{21}$ of making the 40 rev.

Split the ratio: $\frac{6}{21}=\frac{12}{21}\times\frac{1}{2}$,

$$\frac{1}{2}\times\frac{24}{24}=\frac{24}{48} \quad \text{gear } A \text{ (driver)}$$
$$\text{gear } B \text{ (driven)}$$

$$\frac{12}{21}\times\frac{2\frac{2}{3}}{2\frac{2}{3}}=\frac{32}{56} \quad \text{gear } C \text{ (driver)}$$
$$\text{gear } D \text{ (driven).}$$

(See Fig. 158.)

139 moves of 12 holes each on a 42-hole circle will give the required divisions, or 6 holes on a 21-hole circle.

138. Milling Spirals. The **lead** of a spiral is reckoned in the same way as the lead of a screw, namely, the distance it would advance in one revolution.

In milling spirals, gears are used to connect the dividing head and the feed screw on the table of the milling

machine, so that the spindle is revolved a definite amount in proportion to the travel of the table.

The gears for spiral milling are arranged as in Fig. 160. The worm gear is connected by a bevel gear with the index plate. If the stop pin is removed from the index plate and the crank pin inserted in any hole, the revolution of the index plate will cause the index crank to rotate and this in turn will rotate the worm in the inside of the index head. The worm gear on the outside of the head is made to rotate by a train of gears connected with the screw gear

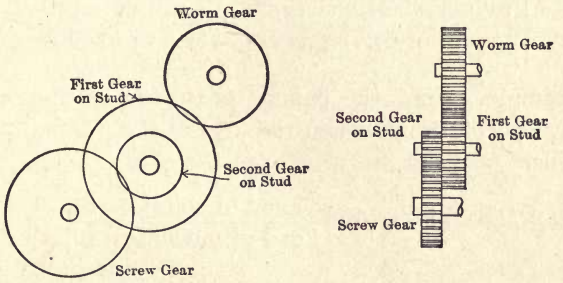


FIG. 160.—Gears for Spiral Milling.

on the feed table. Before calculating the change gears for cutting a spiral, it is necessary to know the *lead of the milling machine.*

139. To Calculate the Lead of a Milling Machine. The lead of a milling machine may be defined as the distance that the table travels for one complete revolution of the spindle. To calculate the lead, first find by trial how many turns it takes of the worm gear to turn the spindle through one complete revolution. Suppose the number is 40. Next find the lead of the feed screw on the table; suppose it is $\frac{1}{4}$ in. If the worm gear and feed screw are

now connected by gears all of the same size, the table will advance $40 \times \frac{1}{4}$ or 10 ins. for one complete revolution of the spindle. The lead of the machine would therefore be 10 ins.

140. To Calculate Change Gears for Cutting a Spiral. The change gears for spiral milling are calculated in the same way as for screw cutting on a lathe. The gears must be arranged so as to obtain a given ratio between the lead of the spiral to be cut and the lead of the machine. The relation between the gears and leads may be arranged in the form of a proportion:

$$\frac{\text{Product of driven gears}}{\text{Product of driving gears}} = \frac{\text{Lead of spiral}}{\text{Lead of machine}}.$$

Example. Find the change gears for cutting a spiral with a lead of 16 ins. when the lead of the milling machine is 10 ins.

Solution. The ratio: $\frac{\text{Lead of spiral}}{\text{Lead of machine}} = \frac{16}{10}$.

Since four gears are to be used we split this ratio:

$$\frac{16}{10} = \frac{4}{5} \times \frac{4}{2}.$$

Multiply by any number to get the gears in stock:

$\frac{4}{5} \times \frac{8}{8} = \frac{32}{40}$ gear on worm (driven)
first gear on stud (driver)

$\frac{4}{2} \times \frac{12}{12} = \frac{48}{24}$ second gear on stud (driven)
gear on screw (driver) (See Fig. 160.)

In this solution the numerators are the driven gears and the denominators are the driving gears. When the lead of

the spiral is greater than that of the machine, the gear on the worm must run more slowly than the screw gear; if the lead of the spiral is less than that of the machine, the worm gear must run more rapidly.

141. To Find the Angle of the Table. In order to have a groove milled in the same shape as the cutter making it, the table of the milling machine must be turned at an angle when a spiral cut is taken.

Example. Find the angle of the milling machine table

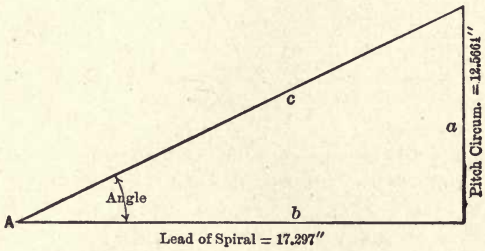


FIG. 161.—Finding Angle of the Table.

when cutting a spiral with a lead of 17.297 ins. on a gear blank 4 ins. in pitch diameter.

Solution. Draw a right-angle triangle with a base equal to the lead of the spiral (17.297 ins), and an altitude equal to the pitch circumference of the gear blank (4×3.1416) as in Fig. 161.

The angle A of this triangle will be the required angle of the table of the milling machine. The value of this angle is easily found by trigonometry (see Chapter XVII).

$$\text{Tangent } A = \frac{a}{b},$$

$$\text{Tangent } A = \frac{12.5664}{17.297} = .7265.$$

From a table of tangents, the value of the angle for a tangent of .7265 is found to be 36°. Ans.

The " hand " of the spiral is determined by the number of gears in the train. Without the idler, the gear train will cause a right-hand spiral to be cut, and with the idler in the train, a left-hand spiral will be cut.

PROBLEMS

1. Find the differential indexing for 101 divisions on an index head with a ratio of 40 to 1.

2. Calculate the differential indexing for 212 divisions on an index head with a ratio of 40 to 1.

3. What is the lead of a milling machine when the thread on the feed screw of the table has a lead of $\frac{1}{4}$ in. and the worm gear makes 60 revolutions to 1 revolution of the spindle?

4. Calculate the change gears for milling a spiral with a lead of 12 ins. on a milling machine which has a lead of 10 ins.

5. Calculate the change gears for milling a spiral with a lead of 21 ins. when the milling machine has a lead of 20 ins.

6. A spiral with a lead of 16.66 ins. is to be cut on a gear blank with a pitch diameter of 4 ins. Calculate the angle for setting the table.

7. Find the angle for setting the table when the spiral lead is 12.167 ins. and the pitch diameter of the blank $3\frac{1}{4}$ ins.

REVIEW PROBLEMS

1. A milling cutter $2\frac{1}{8}$ ins. in diameter is running at a cutting speed of 42 ft. per minute. Find its number of revolutions per minute.

2. The diameter of a milling cutter is 3 ins. How many R.P.M. should it run for a cutting speed of 50 ft. per minute?

3. What indexing would you use by the direct method for 3 equally spaced cuts on a round bar?

4. Calculate the indexing for 56 divisions when the dividing head ratio is 60 to 1.

5. Calculate the indexing for 72 divisions when the ratio of the dividing head is 40 to 1.

6. Calculate the change gears for differential indexing of 159 divisions on a dividing head with a ratio of 40 to 1.

7. A milling machine has a quadruple screw with a pitch of $\frac{1}{4}$ in. If the worm gear turns 40 times to 1 turn of the spindle, what is the lead of the machine?

8. Calculate the change gears for differential indexing of 187 divisions when the ratio of the dividing head is 40 to 1.

9. At what angle should the table of a milling machine be set, when the lead of the spiral is 19.04 ins. on a blank $4\frac{1}{4}$ ins. in diameter?

10. The following change gears were used in cutting a spiral: drivers, 48, 40; driven, 32, 72. If the lead of the machine was 20 ins., what was the lead of the spiral?

11. The table of a milling machine is set at an angle of 22°. (Tangent = .404.) If the lead of the spiral is 31.12 ins., what is the pitch diameter of the blank?

12. A milling cutter running at 80 R.P.M. has a cutting speed of 39.3 ft. per minute. Find the diameter of the cutter.

13. The feed screw on the table of a milling machine has a lead of $\frac{1}{4}$ in. How many R.P.M. must it turn for a feed of $1\frac{7}{8}$ ins. per minute?

14. The following change gears are used in milling a spiral: drivers, 72, 32; driven, 40, 64. If the lead of the spiral is 22.3 ins., what is the lead of the machine?

15. With a spiral lead of 24 ins., what is the angle of setting of the table for a 3-in. diameter blank?

CHAPTER XVI

VOLUME AND WEIGHT

142. Definitions. The **volume** of a body is the amount of space it occupies, or it is often spoken of as the cubical contents of the body.

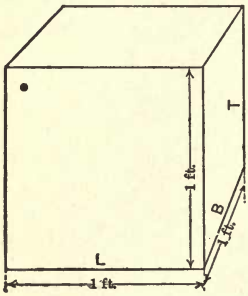


FIG. 162.—A Cube.

Three dimensions are found in a solid body, as length, L, breadth, B, and thickness, T, in Fig. 162. If these three dimensions are all equal, the solid is known as a **cube**. It is customary in a rectangular block to consider the longest side as the length of the block, the next longest as the breadth, and the smallest side as the thickness.

143. To Find the Volume of a Rectangular Solid. The square iron bar shown in Fig. 163 has an area 3×3 ins. at one end. A layer 1 in. deep on this end of the bar is made up of 9 little cubes, each containing 1 cu.in. of material. The whole bar contains 10 such layers, so that the total number of cubic inches in the bar is 9×10 or 90 cu.ins.

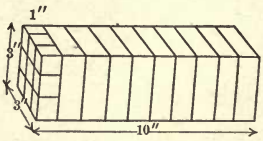


FIG. 163.—A Rectangular Solid.

The same result is obtained by multiplying the length

of the bar (10 ins.) by its breadth (3 ins.) by its thickness (3 ins.).

$$10 \times 3 \times 3 = 90 \text{ cu.ins.}$$

The volume of any rectangular solid may be found in this way:

$$Volume = length \times breadth \times thickness.$$

144. Weight of Square Iron Bars. 1. One cubic inch of wrought iron weighs .2778 lb. A bar 1 ft. long and 1 in. square at the end, Fig. 164, will weigh $.2778 \times 12 = 3.33$ lbs.

From this it is easy to reckon the weight of a bar in terms of its length. If a bar 1 ft. long weighs 3.33 lbs., a bar 5 ft. long will weigh $3.33 \times 5 = 16.65$ lbs.

2. If a bar is 2 ins. square, the area of the end will be 4 sq.ins. and a piece 1 ft. long will weigh:

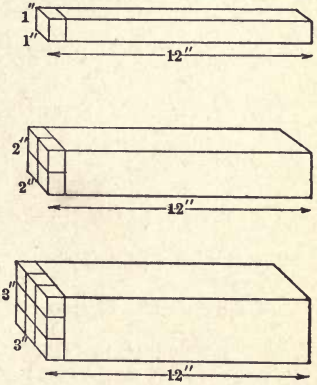


FIG. 164.—Square Bars.

$$.2778 \times 4 \times 12 = 13.33 \text{ lbs.}$$

3. If a bar is 3 ins. square, the area of one end will be 9 sq.ins. and a piece 1 ft. long will weigh:

$$.2778 \times 9 \times 12 = 30.00 \text{ lbs.}$$

The weight of 1 ft. of a square bar of any size may be found thus:

$$Weight\ of\ 1\ ft. = .2778 \times area\ of\ end \times 12.$$

Steel bars are about 2% heavier than wrought iron.

145. Weight of Round Iron Bars. The weight of 1 ft. of a round iron bar, Fig. 165, may be found in the same manner as for a square bar, the only difference being that the area of the end of the bar is the area of a circle, whose diameter is given. For a bar 1 in. in diameter and 1 ft. long the weight will be:

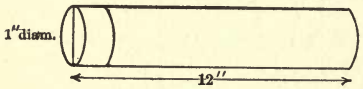


FIG. 165.—Round Bar.

$$\textit{Weight of } 1 \textit{ ft.} = .2778 \times (1^2 \times .7854) \times 12 = 2.61+ \textit{ lbs.}$$

146. Weight of Sheet Iron. A square iron plate 1 in. thick and measuring 1 ft. on each side, Fig. 166, contains:

$12 \times 12 \times 1 = 144$ cu.ins.

The weight of this is:

$.2778 \times 144 = 40$ lbs.

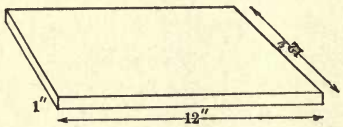


FIG. 166.—Iron Plate.

The weight of a square foot of iron of any thickness will be in proportion to its thickness.

Example. What is the weight of 1 sq.ft. of sheet iron .032 in. thick (No. 20 gauge)?

Solution. One square foot of iron 1 in. thick weighs 40 lbs.

Then $40 \times .032 = 1.28$ lbs. weight. Ans.

$$\textit{Weight of } 1 \textit{ sq.ft. sheet iron} = 40 \times \textit{thickness in thousandths inch.}$$

147. Relative Weight or Specific Gravity. The specific gravity of a substance is the number of times it is as heavy

as some standard substance. Water is usually taken as the standard.

In order to compare the weights of two substances we must take an *equal volume* of each. One cubic foot of water weighs approximately 62.5 lbs. and 1 cu.ft. of iron weighs approximately 480 lbs. The *number of times* that the iron is heavier than the water is $480 \div 62.5 = 7.7$ (nearly). The specific gravity of iron is, therefore, represented by the number 7.7.

148. Table of Specific Gravities. Taking water as a standard, that is, calling its specific gravity 1, the relative weights or specific gravities of other substances are given in the table.

Material.	Specific Gravity.
Aluminum (cast)	2
Brass (cast)	7.8 to 8.4
Brass (rolled)	8.4
Bronze	8.5 (about)
Copper (rolled)	8.8 to 9
Iron (cast)	6.9 to 7.5
Iron (wrought)	7.7
Lead	11.38
Steel	7.85
White pine	.41
Mahogany	.56
Birch	.67

The weight of a substance may be found by means of its specific gravity, if the volume of the substance is known.

Example. What is the weight of a cast-iron block 3 ft. long, 18 ins. wide and 9 ins. thick, if its specific gravity is 7.5?

Solution.

Volume $3 \times 1\frac{1}{2} \times \frac{3}{4}$ ft. $= 3\frac{3}{8}$ cu.ft.;

Weight of 1 cu.ft. of water $= 62.5$ lbs.

Weight of 1 cu.ft. of cast iron $= 62.5 \times 7.5 = 468.75$ lbs.

Weight of block $= 468.75 \times 3\frac{3}{8} = 1582+$ lbs. Ans.

149. Weights of Castings. 1. *By Volume.* The volume of small patterns may be quickly found as shown in Fig. 167.

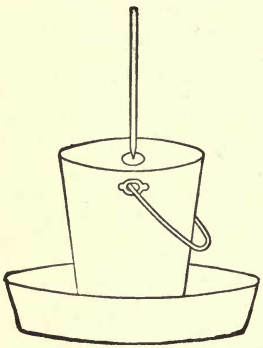


FIG. 167.—Finding Volume of a Small Pattern.

Place a pail in a large shallow pan and fill the pail *even full* of water, taking care that none runs over into the pan. Now place the pattern carefully in the water until it is entirely under water. If the pattern is made of wood it may be pushed under with a wire or sharp-pointed instrument; only the pattern should be under water, not the hands or anything used to hold the pattern. The amount of water which overflows into the pan will be equal to the volume of the pattern. This water can be measured by the quart. One quart of water contains 57.75 cu.ins. and weighs 2.08 lbs.

Example. The volume of a pattern was found to be $1\frac{3}{4}$ quarts. If the casting of it is to be made of brass (specific gravity $= 8$), what will the casting weigh?

Solution.

$2.08 \times 1\frac{3}{4} = 3.64$ lbs. weight of water.

$3.64 \times 8 = 29.12$ lbs. weight of casting. Ans.

2. *By Comparative Weight.* When the volume of the pattern cannot be easily determined, it may be weighed

on a pair of scales, and the weight of the casting to be made may be found by comparing the specific gravity of the pattern with that of the metal used for the casting.

Example. A pattern made of white pine weighs 3 lbs. What will be the approximate weight of the piece of cast iron made from it, if the white pine has a specific gravity of .41 and the cast iron a specific gravity of 6.9?

Solution. The number of times that cast iron is heavier than pine is:

$$6.9 \div .41 = 16.8 \text{ times.}$$

$$\text{Weight of casting} = 16.8 \times 3 = 50.4 \text{ lbs.} \qquad \text{Ans.}$$

The weight of core prints should not be included in the weight of patterns. When a casting is to be made of the same kind of metal as the pattern, there is a loss of about 5% in the weight of the casting.

PROBLEMS

1. Find the volume of a steel bar $2\frac{1}{2}$ ft. long, $4\frac{1}{2}$ ins. wide and $1\frac{1}{2}$ ins. thick.
2. What is the approximate weight of a steel bar $3\frac{1}{2}$ ins. square and 5 ft. long?
3. A round bar of wrought iron is 8 ft. long and $1\frac{3}{4}$ ins. in diameter. Find its weight.
4. Calculate the weight of a square bar of wrought iron measuring $2\frac{5}{8}$ ins. on a side, 6 ft. in length.
5. Find the weight of a round steel bar 8 ft. long and $2\frac{1}{16}$ ins. in diameter.
6. Find the weight of a piece of wrought iron 24 by 36 ins., if it is .0253 in. thick.
7. A piece of steel shafting is $2\frac{1}{8}$ ins. in diameter and 18 ft. long. Calculate its weight.
8. A square bar of wrought iron 10 ft. long weighs 75 lbs. What is the size of the end of the bar if 1 cu.in. of iron weighs .2778 lb.?

9. An engine base made of cast iron weighed 160 lbs. How much less would it weigh if made of aluminum, if the specific gravity of the cast iron is 7.2 and of aluminum 2?

10. Find the weight of a steel plate 72 by 36 ins. if it is .3249 in. thick.

11. A cast-iron tank is 3 ft. long, $2\frac{1}{2}$ ft. wide and $2\frac{3}{4}$ ft. deep, outside measurements. If the tank is $\frac{1}{2}$ in. thick and the iron weighs .2778 lb. per cubic inch, what is the weight of the tank?

12. A round wrought-iron bar is 4 ft. long and weighs 64 lbs. Find the diameter of the bar, when 1 cu.in. weighs .2778 lb.

13. Find the weight of a circular sheet of steel 3 ft. in diameter and .1019 in. thick.

14. Calculate the amount of stock to be used in making 100 stud bolts each $\frac{7}{8}$ in. in diameter and $3\frac{1}{2}$ ins. long, allowing $\frac{1}{8}$ in. for cutting off each bolt.

15. A wooden pattern weighs 8 lbs. If an aluminum casting is to be made from it, what will the casting weigh when the specific gravity of the wood is .41 and of aluminum 2?

CHAPTER XVII

SHOP TRIGONOMETRY

150. Definitions. Trigonometry shows how to find the value of an unknown part of a triangle when the values of the other parts are given. It is based on the fact that there are definite relations between the angles and sides of a triangle.

A **function** is a quantity whose value depends on other quantities.

In Fig. 168, AL and AH are the sides of the angle A.

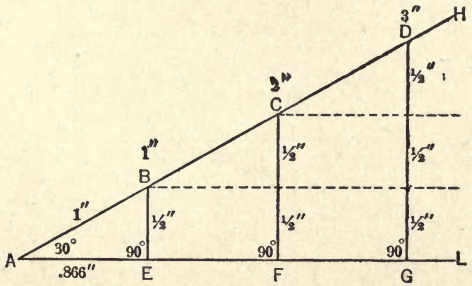


FIG. 168.

Suppose distances of 1 in., 2 in. and 3 in. are marked off from A on the line AH, and perpendiculars are then dropped from points B, C, and D, to the line AL. The perpendicular CF will be found to be twice as long as BE, and perpendicular DG will be three times as long.

If we consider the triangles ABE, ACF and ADG, a similar relation exists between the lengths of the sides BE and AB, AE and AB, etc. These relations may be shown by numbers such as:

$$\frac{BE}{AB}=\frac{\frac{1}{2}}{1}=.5; \quad \frac{AE}{AB}=\frac{.866}{1}=.866, \text{ etc.}$$

In any right triangle there are *six* such relations, which are the *functions* of some angle of the triangle, such as angle A. For angles of the *same size* (same number of degrees) these relations have the same respective values whether the triangle is large or small.

151. Common Functions of Right Triangle. Of the

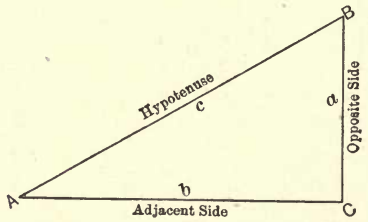


FIG. 169.

six functions of the angle A in Fig. 169, only three are of importance to the machinist. These three functions are very useful in the calculation of shop problems and are stated as follows:

1. $\frac{\text{Opposite side}}{\text{Hypotenuse}}=\frac{a}{c}$ known as the **sine** of angle A.

2. $\frac{\text{Adjacent side}}{\text{Hypotenuse}}=\frac{b}{c}$ known as the **cosine** of angle A.

3. $\frac{\text{Opposite side}}{\text{Adjacent side}}=\frac{a}{b}$ known as the **tangent** of angle A.

152. Trigonometric Tables. If we take a circle with a radius of 1 in., values of the functions may be worked out numerically for the different angles and arranged in tabular form. Fig. 170 shows the functions for an angle of 30°.

Sine of angle $A = \frac{a}{c} = \frac{\frac{1}{2}}{1} = .5$, written, $\sin 30° = .5$;

Cosine of angle $A = \frac{b}{c} = \frac{.866}{1} = .866$, written, $\cos 30° = .866$;

Tangent of angle $A = \frac{a}{b} = \frac{\frac{1}{2}}{.866} = .577$, written, $\tan 30° = .577$.

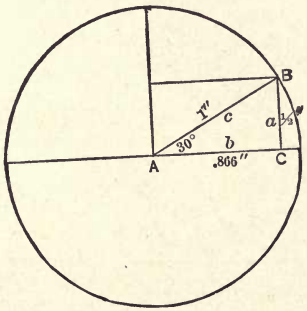


FIG. 170.

If the circle is drawn with a larger radius than 1 in., the sides of the triangle will all be longer in the same proportion, therefore the ratios between these sides will have the same values as before. The table of values for all angles from 0° up to 90° is given on page 195.

153. To Find the Function of an Angle. Example 1. What are the sine, cosine and tangent of an angle of 29°?

In the table on page 195, column 1, find 29°. Reading across the page; sin = .485, cos = .875 and tan = .554. Ans.

Example 2. What are the sine, cosine and tangent of an angle of 50° 30′ (30 minutes)?

From the table: $\sin 55° = .819$

$\sin 56° = .829.$

The sine of 55° 30′ will come between these values. It may be found thus:

Difference between 55° and 56° = 1° or 60 minutes;

Difference between sines = .829 − .819 = .010.

Then $60' = .010$

$$30' = \frac{30}{60} \text{ of } .010 = .005.$$

Therefore: $55° = .819$

$30' = .005$

$55° 30' = .824.$ Ans.

154. To Find the Angle from its Function. Example 1. The tangent of an angle is 1.73. What is the angle?

In the table on page 195, last column, find 1.73. Following back across the page, the angle is 60°. Ans.

Example 2. The cosine of an angle is .967. What is the angle?

From the table:

The nearest value to .967 in the cosine column is .966,

THREE-PLACE NATURAL TRIGONOMETRIC FUNCTIONS

Deg.	sin	cos	tan	Deg.	sin	cos	tan
0	.000	1.000	.000	46	.719	.695	1.04
1	.017	.999	.017	47	.731	.682	1.07
2	.035	.999	.035	48	.743	.669	1.11
3	.052	.999	.052	49	.755	.656	1.15
4	.070	.998	.070	50	.766	.643	1.19
5	.087	.996	.087	51	.777	.629	1.23
6	.105	.995	.105	52	.788	.616	1.28
7	.122	.993	.123	53	.799	.602	1.33
8	.139	.990	.141	54	.809	.588	1.38
9	.156	.988	.158	55	.819	.574	1.43
10	.174	.985	.176	56	.829	.559	1.48
11	.191	.982	.194	57	.839	.545	1.54
12	.208	.978	.213	58	.848	.530	1.60
13	.225	.974	.231	59	.857	.515	1.66
14	.242	.970	.249	60	.866	.500	1.73
15	.259	.966	.268	61	.875	.485	1.80
16	.276	.961	.287	62	.883	.469	1.88
17	.292	.956	.306	63	.891	.454	1.96
18	.309	.951	.325	64	.899	.438	2.05
19	.326	.946	.344	65	.906	.423	2.14
20	.342	.940	.364	66	.914	.407	2.25
21	.358	.934	.384	67	.921	.391	2.36
22	.375	.927	.404	68	.927	.375	2.48
23	.391	.921	.424	69	.934	.358	2.61
24	.407	.914	.445	70	.940	.342	2.75
25	.423	.906	.466	71	.946	.326	2.90
26	.438	.899	.488	72	.951	.309	3.08
27	.454	.891	.510	73	.956	.292	3.27
28	.469	.883	.532	74	.961	.276	3.49
29	.485	.875	.554	75	.966	.259	3.73
30	.500	.866	.577	76	.970	.242	4.01
31	.515	.857	.601	77	.974	.225	4.33
32	.530	.848	.625	78	.978	.208	4.70
33	.545	.839	.649	79	.982	.191	5.14
34	.559	.829	.675	80	.985	.174	5.67
35	.574	.819	.700	81	.988	.156	6.31
36	.588	.809	.727	82	.990	.139	7.12
37	.602	.799	.754	83	.993	.122	8.14
38	.616	.788	.781	84	.995	.105	9.51
39	.629	.777	.810	85	.996	.087	11.4
40	.643	.766	.839	86	.998	.070	14.3
41	.656	.755	.869	87	.999	.052	19.1
42	.669	.743	.900	88	.999	.035	28.6
43	.682	.731	.933	89	.999	.017	57.3
44	.695	.719	.966	90	1.000	.000	Inf.
45	.707	.707	1.000				

and the next value above is .970. The angle lies between 14° and 15°.

Difference between the values = .970 − .966 = .004;
Difference between given value and lower value:

$$.967 - .966 = .001.$$

Given value is $\frac{.001}{.004}$ or $\frac{1}{4}$ the distance between the table values, or $\frac{1}{4}$ away from 15° toward 14°. This is $\frac{3}{4}$ away from 14°.

Whole value between degrees 60′,

$$\tfrac{3}{4} \text{ of } 60 = 45'.$$

Required angle is 14° 45′. Ans.

PROBLEMS

1. Find the sine, cosine and tangent for an angle of 35°.
2. Find the tangent for an angle of 29° 15′.
3. What is the cosine for an angle of 45° 45′?
4. How do the sine and cosine for an angle of 45° compare in value?
5. The tangent of an angle is .74. What is the angle?
6. The sine of an angle is .249. What is the angle?
7. The cosine of an angle is .69. What is the angle?
8. What is the sine of an angle of 22° 30′?

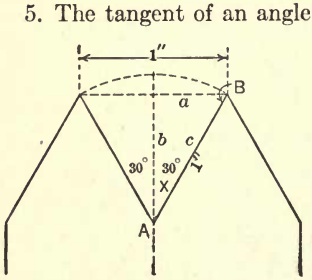


Fig. 171.—Finding Depth of a Thread.

155. To Find the Depth of a 60° Thread. *A. When the Pitch is* 1 *in. First Solution.* Let the pitch be 1 in. A line drawn from the bottom of the thread perpendicular to a line across the top, Fig. 171,

will form a right triangle at each side of the thread. The whole angle at A is divided into two equal parts, each 30°. The depth of the thread is represented by the line b. In the right triangle marked a, b, c, in the figure, we have:

$$\frac{b}{c} = \cos x.$$

By algebra, $b = c \times \cos x.$

Substituting values: $b = 1 \times \cos 30°;$

$b = 1 \times .866$ (from table);

$b = .866''.$ Ans.

Second Solution.

Angle $B = 60°;$

$$\frac{b}{c} = \sin B.$$

By algebra, $b = c \times \sin B.$

Substituting values: $b = 1 \times \sin 60°;$

$b = 1 \times .866;$

$b = .866''.$ Ans.

B. When the Pitch is Some Other Number. Solution. Let the pitch be $\frac{1}{8}$ in. Here the value of the line c is $\frac{1}{8}$ in. By drawing a perpendicular from the bottom of the thread to a line crossing the top, Fig. 172, the angle x is 30°.

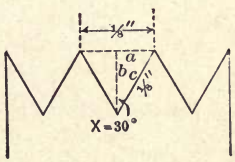


FIG. 172.

Then: $$\frac{b}{c} = \cos x.$$

By algebra, $b = c \times \cos x$.

Substituting values: $b = \frac{1}{8} \times \cos 30°$;

$b = \frac{1}{8} \times .866$;

$b = .108''$. Ans.

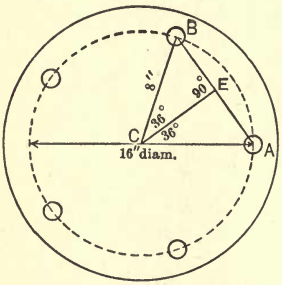


FIG. 173.—Cylinder Head.

156. To Space Holes. It is required to space five holes at equal distances around the cylinder head shown in Fig. 173. The distance AB is to be found for spacing the holes. Since there are 360 degrees in a circle, the whole angle at C is $360 \div 5 = 72°$.

In the triangle ACB:

$CA = CB$ (each is a radius of the circle).

A line drawn from C perpendicular to line AB will divide the angle C into two equal parts, also the line AB into two equal parts.

Then $\frac{1}{2}$ angle $C = 72 \div 2 = 36°$.

In the small right triangle ECB:

$$\frac{EB}{CB} = \sin 36°.$$

By algebra, $EB = CB \times \sin 36°$.

Substituting values: $EB = 8 \times .588$;

$$EB = 4.704''.$$

The distance AB is twice $EB = 4.704 \times 2 = 9.408''$. Ans.

157. To Find the Angle of Taper. The **angle of taper** is the amount the taper inclines to the axis of the piece of work.

Example. Find the angle of taper of the piece of work shown in Fig. 174.

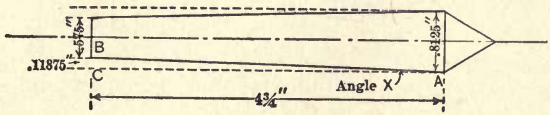


FIG. 174.—Finding the Angle of Taper.

Solution. Large diameter = .8125 in.

Small diameter = .5750

Difference = .2375 in.

Distance $BC = .2375 \div 2 = .11875$ in.

$$\frac{BC}{AC} = \tan x.$$

Substituting values: $\frac{.11875}{4.750} = \tan x.$

$$.025 = \tan x.$$

From table: $\tan 1° = .017$

$\tan 2° = .035$

The angle is between 1° and 2°.
Difference between values = .035 − .017 = .018.
Diff. between given value and lower value = .025 − .017 = .008.

Given value is $\frac{.008}{.018}$ or $\frac{4}{9}$ the distance between, or away from 1°; $\frac{4}{9}$ of 60 = 27′.

Required angle is 1° 27′. Ans.

158. To Find Greatest Square in Round Stock. Example. What is the length of side of the greatest square that can be made from a piece of round stock 2 ins. in diameter?

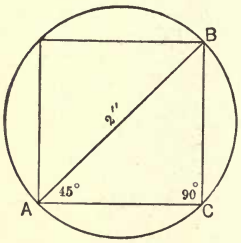


FIG. 175.

Solution. In Fig. 175 the diagonal of the square to be made is the diameter of the piece of round stock. The triangle ABC is equal to $\frac{1}{2}$ of the square. The angle at A is 45°.

In the triangle ABC:

$$\frac{CB}{AB} = \sin A.$$

By algebra, $CB = AB \times \sin A.$

Substituting values: $CB = 2 \times \sin 45°;$

$$CB = 2 \times .707;$$

$CB = 1.414$ ins. side of square. Ans.

PROBLEMS

1. A square bar of iron measures 3 ins. on each side. What is the length of the diagonal across the end of the bar?

2. Calculate the size of the largest square that can be milled on a piece of round stock $2\frac{5}{8}$ ins. in diameter.

3. What distance would be used in a straight line between holes for spacing 5 bolts on a circle 9 ins. in diameter?

4. Find the apothem of a hexagon measuring 5 ins. on a side.

5. What is the smallest diameter of round stock on which a square measuring $2\frac{53}{64}$ ins. on a side may be made?

6. Find the angle of taper on a piece of work which tapers .5 in. to a foot.

7. Show that $\sin A = \cos B$ for any angle in a right triangle.

8. Find the angle of taper on a piece of work which tapers $\frac{3}{4}$ in. to a foot.

9. A circle 16 ins. in diameter has 20 holes spaced equally around it. Find the straight-line distance between any two holes.

10. Find the apothem of a hexagon measuring $2\frac{7}{8}$ ins. on a side.

CHAPTER XVIII

BUSINESS ORGANIZATION

159. Capital. The purpose of this chapter is to outline the organization of the business as a whole. All business must be started with capital, which may be most easily defined as the money invested in a business. Capital is of little use while it is idle in a safe or strong box, and earns only a small amount of interest when deposited in a bank. In order to earn more money, capital may be invested in some way, such as in a foundry or machine shop.

160. Plant and Machinery. To carry on a manufacturing business we must first have a place in which to work and machinery to help in the work. Some of our capital is therefore invested in a building and machinery. This is merely an exchange of money values and no new product has as yet resulted from our investment.

161. Labor. Work at a machine with one's hands we call *labor*. In order that the business may now produce something, we purchase raw material, such as iron, steel, etc., with a part of our capital, and hire labor, i.e., the time and efforts of individual workmen, to convert this raw material into a finished product. This product can be sold at a profit, which is the return on our investment of capital.

162. Accounting. To guard against losing any of our capital, it becomes necessary to keep a record of all our

transactions in business. Keeping this record is known as accounting. Business is really a summation of different processes: there is the *receiving* of the raw material; the *production* of the shop; the *sales department*, which sells the goods and converts the production back into money capital; the *office proper*, which may include the *accounting*, *financial* and *executive* part of the business. The shop mechanic seldom stops to consider that the other departments are just as necessary to the business of the company as his own work in the shop. Every piece of raw material that is purchased, every man that is hired and every transaction of the business must be recorded by someone, in order to determine whether the business is profitable or not.

163. Cost to Manufacture an Article.

A. Raw Material+Direct Labor=First Cost. The cost of raw material is not within our control, except the item of receiving goods, i.e., freight, cartage to the shop, etc. Any wages paid to receiving clerks, expense of keeping horses or automobiles and other like items must be added to the cost of raw material in such a way that each department shall have its share of the cost.

Direct labor must also be charged to the proper departments, such as foundry, machine shop, pattern shop, paint shop, etc.

B. First Cost+Direct Factory Expense=Factory Cost. Direct factory expense consists of such items as rent of factory, light, heat, machine repairs, maintenance, depreciation on buildings and equipment. For depreciation, an amount should be set aside each year to accumulate into a sum large enough to equal the original cost of machinery, etc., at a time when they become useless to the company and must be replaced. Allowing for depreciation is simply

providing for a "rainy day," when some outlay must be met. All these items of expense apply to the factory as a whole.

C. Factory Cost+General Expense=Cost to Make. General expense includes those items which apply to the general office or plant and are not properly chargeable to the factory, such as taxes, insurance, interest on borrowed money, maintenance of storehouses for finished product. These items are for the business as a whole and are not easily classified as belonging to any department. They constitute the general or "overhead" expense.

D. Cost to Make+Selling Expense=Cost to Make and Sell. Selling expense is sometimes one of the greatest items of cost and must not be forgotten in estimating the cost of an article. It includes salaries and expenses of salesmen, advertising, free samples and allowances made to agents. The selling expense must not bear too great a proportion of the cost, or the article will become too expensive for people to buy.

E. Cost to Make and Sell+Profit=Price. Profit may be considered the "acid test" of business. It answers the question of the worth of the business, as to whether there is a fair rate of return on the money invested. The profit may be reckoned on the price *per article* manufactured, or it may be reckoned as a percentage on the sum of all the other items.

164. The Workman. Let us consider now the relation of the workman to the business organization. John Jones is hired by the company and is assigned to some foreman. He immediately represents an investment of the capital of the business in that he has contracted to give his services to the company for a certain wage; he becomes a factor of our labor account.

Suppose he is given a piece of steel to machine. This steel represents an investment of our capital in raw material, which after a certain time becomes a finished product through the efforts of Jones and others. It is necessary for us to keep an account of the labor. This is most easily done by the use of *job tickets* and *time cards*.

A. Job Tickets. We may dispense with the actual writing of Jones' name in a book and the statement that he arrived at 7 A.M., worked until 10 o'clock on a lathe, ran a milling machine until 12, and from 1 to 5 o'clock was occupied in assembling work. It is much easier for the record keeper or cost clerk to give a *job number* to each piece of *work* that is being done, and then subdivide each process of its manufacture on a card. Opposite each process a number or punch mark is placed for each workman on the job, with a place for the number of hours each man worked. The total number of hours multiplied by the number of cents paid per hour to each man will give the cost of direct labor on the job. This job ticket is the labor account of the article made.

B. Time Cards. As far as Jones is personally concerned, the total number of hours he worked on all jobs, plus idle time not recorded on any particular job, should equal his total time between punching the time clock at 7 A.M. and leaving at 5 P.M. This clock record is the personal time card for Jones. The form of job ticket and the form of time record will vary in different shops, but the principle is the same in all.

165. Systems of Payment. No matter how finely equipped a factory may be, nor how skillfully its product is advertised, the real success of its business will depend to a large extent upon the character of its workmen and

their interest in their work. Many attempts have been made to devise a system of payment that will be fair to owner and workman. Let us briefly consider some of the well-established systems of paying wages.

A. Pay by the Day. This system is the one most generally used in all lines of business because it requires but little effort to make up a pay roll. Jones works 6 days of 8 hours each at $37\frac{1}{2}$ cents an hour, so his pay at the end of the week will be $18.00. This system can be used in shop, office, shipping room and all through the factory. The chief objection to this plan is that it gives no incentive to the best man to work any harder than the poorest man, as all are paid equally for the same class of work. The employer is always under the general expense of running the plant, so if the men "soldier" on the job, the employer loses just so much money, because some of his capital is non-productive, and the cost of manufacturing an article becomes higher.

B. Piece Work. The fundamental idea of piece work is to "pay according to the results produced." The workman is paid at so much per piece of material finished. This gives the man who is quick with his hands an advantage over the slower workman. In some cases it becomes necessary to cut the price per piece so that a fair average for all may be obtained. This cut often works against the faster workman.

The piece-work plan is also a disadvantage to the employer in that some will try to keep up with the faster ones and begin to be careless so that faulty work is turned out. Because faulty goods are returned to the factory, a system of inspection must be established to check up the work, and this is an additional expense. In order to deal fairly with the employer and employee, the piece-work

plan must be very carefully tested before any definite rate of pay per piece is fixed upon.

C. Differential Rate of Pay. Many of the evils of the piece-work system are remedied by having a differential rate of pay. A certain standard is set for each workman for which he receives a fixed rate of pay. If the average number of pieces turned out in the department is 25 per man, and the day wage is $2.50, the rate of pay is fixed at 10 cents each for finishing 25 pieces. In order to encourage the fast workman, the rate per piece is increased say to 11 cents if he turns out 30 to 34 pieces. On the other hand the slow workman is not paid quite as much, say 9 cents per piece if he turns out only 20 to 24 pieces. To guard against paying too low a rate, there is usually a smallest day wage established so that any workman may earn a living.

This is a fair arrangement for both employer and employee, for one gets the benefit of increased production, and the other gets more pay for more work. That the workman who gets the higher rate of pay is really a cheaper man for the company to hire is shown by the following illustration:

	Articles made.	Labor cost.	Material cost.	General expense.	Total.	Cost per article.
Slow workman..	20 at 9c.	$1.80	$ .40	$1.35	$3.55	$ .17¾
Fast workman...	35 at 11c.	3.85	.70	1.40	5.95	.17

D. Premium System. In this system a premium or bonus is placed on *time saved* rather than an increase made in the price per piece. If a man can do a piece of work in 10 hours for which he receives $3.00, the employer agrees that if he will do the same work in 9 hours he will pay the same rate as for 10 hours, but for the 10th hour of work he will pay a higher rate, say ½ more than the hour rate.

Illustration.

PAY ON THE OLD BASIS

9 hours $\times .30 = \$2.70$
1 hour $\times .30 = \quad .30$

Total $= \$3.00$ per day.

Making 100 articles at $ .03 each.

PAY ON THE NEW BASIS

Saving 1 hour: 9 hours $\times .30 = \$2.70$
1 hour $\times (30+15) = \quad .45$

Total $= \$3.15$ per day.

Making 110 articles at $ .0286 each.

Saving 2 hours: 8 hours $\times .30 = \$2.40$
2 hours $\times (30+15) = \quad .90$

Total $= \$3.30$ per day.

Making 120 articles at $ .0275 each.

E. Profit Sharing. This plan requires a lot of accounting and record keeping. An inventory of the business is made at the end of certain periods and the profit determined. The workmen are given a share of this profit, sometimes as cash in the pay envelope. Suppose $100,000 is figured as the profit for the period and the workmen are to have one-half for their share. If there are 1000 workmen, each would get $\frac{1}{1000}$ of $50,000 or $50, if the division is made equally. This may be modified by dividing the profits according to the relative skill of the workmen,

or according to the length of service, thereby putting a premium on loyalty to the company. Sometimes shares of stock in the company are given in place of cash.

PROBLEMS

1. A man is employed in a factory at $37\frac{1}{2}$ cents per hour, 8 hours per day. What does he earn in a year of 300 working days?

2. If 100 men are employed at \$3.00 per day and have a 10% raise, what will be the extra cost to the employer for a year?

3. A factory employs 200 men for a 10-hour day at 25 cents per hour. If each man wastes 6 minutes a day, calculate the loss to the employer for a month of 26 working days.

4. A man employed at \$3.60 per day of 8 hours wastes 10 minutes each day. How much is his wasted time worth in a year?

5. A man working at \$3.20 per day makes 40 pieces. When put on piece work at the same job, he makes 46 pieces a day at 8 cents per piece. Find the per cent of increase in his wages.

6. The following is a record of work for a week:

	A.M.	P.M.	
Monday:	7.00–12.00	1.00–6.00	(1 hour overtime)
Tuesday:	7.00–12.00	1.00–3.00	(left early; deduct pay)
Wednesday:	7.15–12.00	1.00–5.00	(15 min. late)
Thursday:	8.00–12.00	1.00–5.00	(1 hour late)
Friday:	7.00–12.00	1.00–5.00	
Saturday:	7.00–12.00	1.00–5.00	

The rate of pay is 35 cents an hour. Lateness has a penalty of 10 cents for each $\frac{1}{4}$ hour; overtime is paid at "time and a half." Find the amount due the workman at the end of the week.

7. A workman makes 11 articles a day at 24 cents each. He is told that he may have 25 cents per article if he makes over 11 per day. If he now makes 14 articles a day, what per cent is his pay increased?

8. Compare the worth of these two men to the company by finding the total cost of making the same article. A makes 17 articles at 15 cents each and *B* makes 23 articles at 16 cents each. The material costs 8 cents per article, and the cost of operating is \$1.00 in each case.

9. In a certain factory the men are paid 30 cents per hour for an 8-hour day. A bonus of $\frac{1}{2}$ the hour rate is paid on every hour saved. Find the wages due in the following case:

Job No. 1, time limit 3 hours; done in 2 hours.
Job No. 2, time limit 4 hours; done in 3 hours.
Job No. 3, time limit 5 hours; done in 3 hours.

10. A man is offered his choice of these systems:

(1) For 15 or under, 12 cents per article; for 20 or over, 14 cents per article.
(2) A straight rate of 38 cents per hour.
(3) 35 cents per hour with $\frac{1}{2}$ rate added for every hour saved.

On trial he finds that he can make 21 articles in 7 hours, time limit being 8 hours on the work. Find the wages he would earn by each plan.

APPENDIX

FORMULAS

A formula is simply a short way of stating some rule of calculation. It is made as simple as possible and written in a convenient form for mathematical use.

The quantities concerned are mostly represented by letters, each letter usually being the first letter of the name of the quantity used. For example, in the law for the inclined plane, $EL = WH$, the meaning is: the *effort* multiplied by the *length* of the plane is equal to the *weight* multiplied by the *height* of the plane. Other letters might have been used to state the same relation, but there is likely to be some confusion unless a careful statement is made in regard to the meaning of each letter. When two quantities used in a formula have names beginning with the same letter, it becomes necessary to use some other letter for one of the quantities. There are some quantities which occur so often in mathematical expressions that they are always represented in the same way by custom; an example of this is the use of the Greek letter π, which stands for the number 3.1416.

Using Formulas. Handbooks are filled with formulas of every kind. The average mechanic does not stop to inquire into their meaning, but is content if he has discovered some way of making use of a few which directly concern him. If a few of the letters are changed about

in any one of these, he often does not recognize it as the same formula in meaning. The aim should be to know just why each letter is in the formula, and how the formula itself can be used to the best advantage. There are two ways of working out the value of a quantity from a formula which may be shown in the following way:

1. *First Method.* Take a formula which is used very frequently in electricity. The relation of the electric current to the pressure and resistance concerned in its flow is written:

$$C=\frac{E}{R} \quad \text{or} \quad \text{Current}=\frac{\text{Electromotive force}}{\text{Resistance}}.$$

There are three quantities concerned, and if any two of them are given we may find the value of the third. Many textbooks give this formula in three different forms which are simply algebraic changes of the original formula. The changing about of the letters in this formula illustrates the first method of handling a given formula.

Take the first form given: $C=\frac{E}{R}$ (Form 1.)

Multiply both sides by R: $C\times R=E$, or $E=C\times R$ (Form 2).

Again: Since $C\times R=E$,

Divide both sides by C: $R=\frac{E}{C}$. (Form 3.)

Having these three forms, we use them as follows:

If the electromotive force and resistance are given in a problem, use form 1 to find the current.

If the current and resistance are given in a problem, use form 2 to find the electromotive force.

If the electromotive force and current are given in a problem, use form 3 to find the resistance.

2. *Second Method.* It is generally shorter to keep the formula as given originally and substitute the given values of the quantities before attempting to change the formula about. After the given values are substituted, it often happens that certain of the quantities will cancel and make the arithmetical work for the solution much easier.

Example. A current of 4 amperes is sent through a coil of wire by an electromotive force of 108 volts. What is the resistance of the wire?

Solution. $C=\frac{E}{R}$.

Substituting known quantities: $4=\frac{108}{R}$;

Multiplying both sides by R: $4R=108$

$R=27$ ohms. Ans.

List of Formulas. The formulas given in the following list are simply for reference, as the meaning of a large number of them has already been given in other parts of this book. No attempt has been made to give the meanings of the letters in each formula as the meaning in most cases is sufficiently clear.

Area of rectangle: Area $=L\times W$.

Area of triangle: Area $=\frac{1}{2}\times B\times H$.

Area of hexagon: Area $=\frac{1}{2}\times P\times$ Apoth

Area of circle: Area $=$ (diam.)$^2\times .7854$.

In a right triangle: $a^2+b^2=c^2$.

Cutting speed: $\text{Speed} = \text{cir.} \times \text{R.P.M.}$

Set over for taper: $\text{Set over} = \frac{L \times T}{2}$.

Double depth "V" thread: $\text{Depth} = \frac{1.732}{N}$.

Double depth U.S. thread: $\text{Depth} = \frac{1.299}{N}$.

Law of lever: $E \times Pa = W \times Wa$.

Law of wheel and axle: $E \times R = W \times r$.

Law of pulley: $E \times n = W$.

Laws of inclined plane: $E \times l = W \times h$.

$E \times b = W \times h$.

Law of screw: $E \times \text{cir.} = W \times ld$.

General law of machines: $E \times Ed = W \times Wd$.

Work: $W = f \times s$.

Power: $P = \frac{f \times s}{t}$.

Horse-power: $\text{H.P.} = \frac{f \times s}{33{,}000 \times \text{min.}}$.

Horse-power of steam engine: $\text{H.P.} = \frac{\text{P.L.A.N.}}{33{,}000}$.

Length of open belt: $L = 3\frac{1}{4}(R+r)+2l$.

Length of crossed belt: $L = 3\frac{3}{8}(R+r)+2l$.

Diametral pitch: $P = \frac{3.1416}{P'}$.

Circular pitch: $P' = \frac{3.1416}{P}$.

Thickness of tooth: $t = \frac{1.57}{P}$.

Whole depth of tooth: $D'' + f = \frac{2}{P} + \frac{t}{10}$.

Outside diameter of gear blank: $\text{O.D.} = \frac{N+2}{P}$.

Number of teeth: $N = (\text{O.D.} \times P) - 2$.

Diametral pitch: $P = \frac{N+2}{\text{O.D.}}$.

Volume of rectangular body: $V = L \times B \times T$.

TABLES

TABLE OF DECIMAL EQUIVALENTS

Of Eighths, Sixteenths, Thirty-seconds and Sixty-fourths of an Inch

$\frac{1}{64}$	.015625	$\frac{33}{64}$	.515625
$\frac{1}{32}$	.03125	$\frac{17}{32}$	.53125
$\frac{3}{64}$	.046875	$\frac{35}{64}$	.546875
$\frac{1}{16}$	.0625	$\frac{9}{16}$	.5625
$\frac{5}{64}$	.078125	$\frac{37}{64}$	.578125
$\frac{3}{32}$	.09375	$\frac{19}{32}$	.59375
$\frac{7}{64}$	.109375	$\frac{39}{64}$	.609375
$\frac{1}{8}$	.1250	$\frac{5}{8}$	.6250
$\frac{9}{64}$	.140625	$\frac{41}{64}$	.640625
$\frac{5}{32}$	.15625	$\frac{21}{32}$	.65625
$\frac{11}{64}$	.171875	$\frac{43}{64}$	.671875
$\frac{3}{16}$	.1875	$\frac{11}{16}$	.6875
$\frac{13}{64}$	.203125	$\frac{45}{64}$	.703125
$\frac{7}{32}$	.21875	$\frac{23}{32}$	.71875
$\frac{15}{64}$	.234375	$\frac{47}{64}$	.734375
$\frac{1}{4}$	.2500	$\frac{3}{4}$	.7500
$\frac{17}{64}$	.265625	$\frac{49}{64}$	.765625
$\frac{9}{32}$	.28125	$\frac{25}{32}$	.78125
$\frac{19}{64}$	.296875	$\frac{51}{64}$	.796875
$\frac{5}{16}$	.3125	$\frac{13}{16}$	.8125
$\frac{21}{64}$	.328125	$\frac{53}{64}$	.828125
$\frac{11}{32}$	.34375	$\frac{27}{32}$	.84375
$\frac{23}{64}$	.359375	$\frac{55}{64}$	.859375
$\frac{3}{8}$	.3750	$\frac{7}{8}$	.8750
$\frac{25}{64}$	.390625	$\frac{57}{64}$	.890625
$\frac{13}{32}$	.40625	$\frac{29}{32}$	.90625
$\frac{27}{64}$	.421875	$\frac{59}{64}$	.921875
$\frac{7}{16}$	.4375	$\frac{15}{16}$	.9375
$\frac{29}{64}$	.453125	$\frac{61}{64}$	.953125
$\frac{15}{32}$	.46875	$\frac{31}{32}$	.96875
$\frac{31}{64}$	.484375	$\frac{63}{64}$	.984375
$\frac{1}{2}$	.5000	1	1.0000

MILLIMETER EQUIVALENTS OF INCHES

Inch.	0	1/16	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4
0	0.0	1.6	3.2	4.8	6.4	7.9	9.5	11.1	12.7	14.3	15.9	17.5	19.1
1	25.4	27.0	28.6	30.2	31.7	33.3	34.9	36.5	38.1	39.7	41.3	42.9	44.4
2	50.8	52.4	54.0	55.6	57.1	58.7	60.3	61.9	63.5	65.1	66.7	68.3	69.8
3	76.2	77.8	79.4	81.0	82.5	84.1	85.7	87.3	88.9	90.5	92.1	93.7	95.2
4	101.6	103.2	104.8	106.4	108.0	109.5	111.1	112.7	114.3	115.9	117.5	119.1	120.7
5	127.0	128.6	130.2	131.8	133.4	134.9	136.5	138.1	139.7	141.3	142.9	144.5	146.1
6	152.4	154.0	155.6	157.2	158.8	160.3	161.9	163.5	165.1	166.7	168.3	169.9	171.5
7	177.8	179.4	181.0	182.6	184.2	185.7	187.3	188.9	190.5	192.1	193.7	195.3	196.9
8	203.2	204.8	206.4	208.0	209.6	211.1	212.7	214.3	215.9	217.5	219.1	220.7	222.3
9	228.6	230.2	231.8	233.4	235.0	236.5	238.1	239.7	241.3	242.9	244.5	246.1	247.7
10	254.0	255.6	257.2	258.8	260.4	261.9	263.5	265.1	266.7	268.3	269.9	271.5	273.1
11	279.4	281.0	282.6	284.2	285.7	287.3	288.9	290.5	292.1	293.7	295.3	296.9	298.4
12	304.8	306.4	308.0	309.6	311.1	312.7	314.3	315.9	317.5	319.1	320.7	322.3	323.8
13	330.2	331.8	333.4	335.0	336.5	338.1	339.7	341.3	342.9	344.5	346.1	347.7	349.2
14	355.6	357.2	358.8	360.4	361.9	363.5	365.1	366.7	368.3	369.9	371.5	373.1	374.6
15	381.0	382.6	384.2	385.8	387.3	388.9	390.5	392.1	393.7	395.3	396.9	398.5	400.0
16	406.4	408.0	409.6	411.2	412.7	414.3	415.9	417.5	419.1	420.7	422.3	423.9	425.4
17	431.8	433.4	435.0	436.6	438.1	439.7	441.3	442.9	444.5	446.1	447.7	449.3	450.8
18	457.2	458.8	460.4	462.0	463.5	465.1	466.7	468.3	469.9	471.5	473.1	474.7	476.2
19	482.6	484.2	485.8	487.4	488.9	490.5	492.1	493.7	495.3	496.9	498.5	500.1	501.6
20	508.0	509.6	511.2	512.8	514.3	515.9	517.5	519.1	520.7	522.3	523.9	525.5	527.0
21	533.4	535.0	536.6	538.2	539.7	541.3	542.9	544.5	546.1	547.7	549.3	550.9	552.4
22	558.8	560.4	562.0	563.6	565.1	566.7	568.3	569.9	571.5	573.1	574.7	576.3	577.8
23	584.2	585.8	587.4	589.0	590.5	592.1	593.7	595.3	596.9	598.5	600.1	601.7	603.2

39.37 inches = 1 m. = 10 dm. = 100 cm. = 1000 mm.

TAP DRILLS

For Machine Screw Taps

Size of Taps.	No. of Threads.	Size of Drills.	Size of Taps.	No. of Threads.	Size of Drills.
2	48	51	12	24	19
2	56	50	13	20	19
2	64	49	13	24	15
3	40	49	14	20	16
3	48	48	14	22	13
3	56	44	14	24	9
4	32	48	15	18	13
4	36	45	15	20	10
4	40	44	15	24	6
5	30	44	16	16	13
5	32	43	16	18	10
5	36	41	16	20	6
5	40	40	16	24	2
6	30	41	17	16	7
6	32	37	17	18	4
6	36	36	17	20	2
6	40	33	18	16	3
7	28	35	18	18	2
7	30	34	18	20	A
7	32	31	19	16	1
8	24	34	19	18	B
8	30	30	19	20	D
8	32	30	20	16	C
9	24	30	20	18	E
9	28	29	20	20	H
9	30	28	22	16	H
9	32	27	22	18	J
10	24	28	24	14	K
10	28	26	24	16	L
10	30	24	24	18	N
10	32	24	26	14	N
11	24	24	26	16	O
11	28	21	28	14	Q
11	30	19	28	16	S
12	20	24	30	14	T
12	22	20	30	16	V

SIZES OF TAP DRILLS

For Taps U. S. Standard Thread

Diameter Tap, in Inches.	Threads per Inch.	Size of Drill.	Diameter Tap, in Inches.	Threads per Inch.	Size of Drill, Inches.
$\frac{1}{4}$	20	$\frac{3}{16}$ in.	$1\frac{1}{4}$	7	$1\frac{5}{64}$
$\frac{5}{16}$	18	C	$1\frac{3}{8}$	6	$1\frac{11}{64}$
$\frac{3}{8}$	16	N	$1\frac{1}{2}$	6	$1\frac{19}{64}$
$\frac{7}{16}$	14	S	$1\frac{5}{8}$	$5\frac{1}{2}$	$1\frac{25}{64}$
$\frac{1}{2}$	13	$\frac{13}{32}$ in.	$1\frac{3}{4}$	5	$1\frac{1}{2}$
$\frac{9}{16}$	12	$\frac{29}{64}$ in.	$1\frac{7}{8}$	5	$1\frac{5}{8}$
$\frac{5}{8}$	11	$\frac{33}{64}$ in.	2	$4\frac{1}{2}$	$1\frac{23}{32}$
$\frac{11}{16}$	11	$\frac{37}{64}$ in.	$2\frac{1}{8}$	$4\frac{1}{2}$	$1\frac{27}{32}$
$\frac{3}{4}$	10	$\frac{5}{8}$ in.	$2\frac{1}{4}$	$4\frac{1}{2}$	$1\frac{31}{32}$
$\frac{13}{16}$	10	$\frac{11}{16}$ in.	$2\frac{3}{8}$	4	$2\frac{1}{16}$
$\frac{7}{8}$	9	$\frac{47}{64}$ in.	$2\frac{1}{2}$	4	$2\frac{3}{16}$
$\frac{15}{16}$	9	$\frac{51}{64}$ in.			
1	8	$\frac{27}{32}$ in.			
$1\frac{1}{8}$	7	$\frac{61}{64}$ in.			

SIZES OF DRILLS FOR PIPE TAPS

Briggs' Standard

Size, Tap, Inch.	Diameter, Drill, Inch.	Size, Tap, Inch.	Diameter, Drill, Inch.	Size, Tap, Inch.	Diameter, Drill, Inch.
$\frac{1}{8}$	$\frac{11}{32}$	$\frac{3}{4}$	$\frac{51}{64}$	2	$2\frac{3}{16}$
$\frac{1}{4}$	$\frac{7}{16}$	1	$1\frac{1}{8}$	$2\frac{1}{2}$	$2\frac{37}{64}$
$\frac{3}{8}$	$\frac{9}{16}$	$1\frac{1}{4}$	$1\frac{7}{16}$	3	$3\frac{13}{64}$
$\frac{1}{2}$	$\frac{45}{64}$	$1\frac{1}{2}$	$1\frac{23}{32}$		

MACHINE AND WOOD SCREW GAUGE

Number of Screw Gauge.	Size of Number in Decimals.	Number of Screw Gauge.	Size of Number in Decimals.	Number of Screw Gauge.	Size of Number in Decimals.
000	.03152	16	.26840	34	.50528
00	.04468	17	.28156	35	.51844
0	.05784	18	.29472	36	.53160
1	.07100	19	.30788	37	.54476
2	.08416	20	.32104	38	.55792
3	.09732	21	.33420	39	.57108
4	.11048	22	.34733	40	.58424
5	.12364	23	.36052	41	.59740
6	.13680	24	.37368	42	.61056
7	.14996	25	.38684	43	.62372
8	.16312	26	.40000	44	.63688
9	.17628	27	.41316	45	.65004
10	.18944	28	.42632	46	.66320
11	.20260	29	.43948	47	.67636
12	.21576	30	.45264	48	.68952
13	.22892	31	.46580	49	.70268
14	.24208	32	.47896	50	.71584
15	.25524	33	.49212		

TABLE OF EMERY WHEEL SPEEDS

Diameter Wheel in Inches.	Rev. per Minute for Surface Speed of 4,000 Feet.	Rev. per Minute for Surface Speed of 5,000 Feet.	Rev. per Minute for Surface Speed of 6,000 Feet.
1	15,279	19,099	22,918
2	7,639	9,549	11,459
3	5,093	6,366	7,639
4	3,820	4,775	5,730
5	3,056	3,820	4,584
6	2,546	3,183	3,820
7	2,183	2,728	3,274
8	1,910	2,387	2,865
10	1,528	1,910	2,292
12	1,273	1,592	1,910
14	1,091	1,364	1,637
16	955	1,194	1,432
18	849	1,061	1,273
20	764	955	1,146
22	694	868	1,042
24	637	796	955
30	509	637	764
36	424	531	637
42	364	455	546
48	318	397	477

TEMPER COLORS OF STEEL

C.	F.	
221	430	Faint yellow
238	460	Straw color
243	470	Dark straw
277	530	Purple
289	550	Blue
293	560	Full blue
304	580	Polish blue
316	600	Dark blue
400	752	Bright red in the dark
474	884	Red hot in twilight
581	1077	Red, visible by day

TWIST DRILL AND STEEL WIRE GAUGE

No.	Size.	No.	Size.	No.	Size.	No.	Size.
1	.2280	16	.1770	31	.1200	46	.0810
2	.2210	17	.1730	32	.1160	47	.0785
3	.2130	18	.1695	33	.1130	48	.0760
4	.2090	19	.1660	34	.1110	49	.0730
5	.2055	20	.1610	35	.1100	50	.0700
6	.2040	21	.1590	36	.1065	51	.0670
7	.2010	22	.1570	37	.1040	52	.0635
8	.1990	23	.1540	38	.1015	53	.0595
9	.1960	24	.1520	39	.0995	54	.0550
10	.1935	25	.1495	40	.0980	55	.0520
11	.1910	26	.1470	41	.0960	56	.0465
12	.1890	27	.1440	42	.0935	57	.0430
13	.1850	28	.1405	43	.0890	58	.0420
14	.1820	29	.1360	44	.0860	59	.0410
15	.1800	30	.1285	45	.0820	60	.0400

INDEX

Accounting, 202.
Acme thread, 108.
Addendum, 160.
Altitude, 51, 52.
Angle, 56, 57.
" , acute, 57.
" , complementary, 67.
" , obtuse, 57.
" , supplementary, 67.
Angle of table, 181.
" of taper, 199.
Apothem, 54.
Arc, 40.
Area of circle, 54-56.
" of figures, 52.
" of hexagon, 54.
" of rectangle, 50.
" of right triangle, 51.
Assembly drawing, 35.

Base, 22, 51.
Belt, crossed, 154.
" , open, 152.
Belting, 152.
" , horsepower of, 155.
" , speed of, 154.
" , thickness of, 154.
Bevel block, 29.
" gears, 167.
" protractor, 58.
Blueprints, 29.
Brake horsepower, 144.
Bronze, 25.
Business organization, 202.

Calipers, 42.
Cancellation, 9.
Capital, 202.
Centering work, 84, 85.
Center line, 31.
Center of arc, 64.
" of circle, 68.
" of line, 64.
Center square, 62.
Change gears, 112.
Circle, 40.
" , division of, 68-70.
Circular pitch, 160, 161.
Circumference, 40, 41.
Clearance, 160.
Commission, 25.
Complementary angles, 67.
Compound, 116.
" geared lathe, 116.
" machines, 139.
" rest, 94.
Construction lines, 32.
Conventions on blueprints, 36, 37.
Cost to manufacture, 203.
Crank shaper, 121.
Cross section, 31.
" " of materials, 36.

Cup center, 85.
Cutting speed, 86-88, 170.
" spiral, 180.

Decimals, 14.
" , addition of, 16.
" , division of, 18.
" , multiplication of, 17.
" , subtraction of, 17.
Dedendum, 160.
Degree, 41.
Denominator, 2.
Depth of gear tooth, 160, 163.
" of thread, 196.
Diameter, 41.
Diametral pitch, 159, 161.
Differential indexing, 175.
" pulley, 140.
" rate of pay, 207.
Dimension lines, 29.
Direct indexing, 172.
" proportion, 147.
Discounts, 26.
Distance between gears, 166.
Dividers, 84.
Dividing sector, 174.
Draughtman's triangle, 57.
Drill-press work, 122.

Efficiency, 26, 27.
Effort, 125.
" arm, 126.
Exponent, 72.
Extremes, 147.

Feed, 88, 170, 171.
" for finishing, 89.
Fixed pulley, 131.
Foot-pound, 143.
Fractions, 1.
Fractions, addition of, 4.
" , decimal, 14.
" , division of, 10.
" , improper, 2.
" , multiplication of, 8.
" , reduction of, 2.
" , subtraction of, 7.
Friction, 138.
Fulcrum, 125.
Function, 191.
Functions of angle, 193, 194.
" of right triangle, 192.

Gain or loss, 24.
Gear blank, 164.
" " , number of teeth on 165.
" " , outside diameter of, 164.
Gear blank, pitch of, 165.
Geared shaper, 122.
Gearing for thread, 114.
Gear progression, 115.
Gears, 158.
" , bevel, 167.
" , mitre, 167.
" , spur, 158.
" , worm, 168.
Gear tooth, depth, 160, 163.
" " , thickness, 160, 162.
Gear train, 112, 148.
General law of machines, 139.

Hermaphrodite calipers, 84.
Hexagon, 54.
Horsepower, 143.
" of steam engine, 144.
Hypotenuse, 80.

Inclined plane, 133.

Inclined plane, law of, 134, 135.
Index crank, 172.
" of power, 72.
" of root, 73.
" plate, 172.
Indexing, 171.
" , differential, 175-178.
" , direct, 172.
" , simple, 172.
Indicated horsepower, 144.
Integer, 2.
Interest, 25.
Intermediate, 113.
Inverse proportion, 148.

Jarno taper, 91.
Job tickets, 205.

Labor, 202.
Lead of lathe, 114.
" of milling machine, 179.
" of spiral, 178.
Left-hand thread, 118.
Left moment, 127.
Lever, 125.
" , first arrangement, 125, 126.
" , second arrangement, 126, 127.
" , third arrangement, 126, 127, 128.
" , law of, 128.
Loss, 24.
Lowest common denominator, 5.
Lowest terms, 2.

Machinist's rule, 1, 39.
Means, 147.
Mechanical advantage, 132, 133.
" efficiency, 138.
Micrometer caliper, 44-46.
Milling spirals, 178.
Mitre gears, 167.
Mixed numbers, 3.
" " , addition of, 6.
" " , division of, 11.
" " , multiplication of, 9.
" " , subtraction of, 7
Mixtures, 25,
Moments, 126.
Morse taper, 91.
Movable pulley, 131.

Necking, 34.
Numerator, 2.

Obtuse angle, 57.
Outside diameter of thread, 98.
" " of gear blank, 161.

Pay by day, 206.
Percentage, 22.
Perimeter, 54.
Perpendicular, 65.
Perpetual motion, 26.
Piece work, 206.
Pinion, 158.
Pitch diameter, 158.
" line, 158.
Planer work, 118.
Power, 143.
Powers of numbers, 72.
Premium system, 207.
Profit sharing, 208.
Prony brake, 144.
Proportion, 146.
Protractor, 57.
Pulley, 130.
" block, 130.

Pulley, fixed, 131.
" , movable, 131.
" , law of, 132.
" train, 150.

Rack, 167.
Radical sign, 73.
Radius, 41.
Rate, 22.
Ratio, 146.
Rectangle, 50.
" , area of, 50.
Relative weight, 186.
Reverse gears, 117.
Right angle, 56.
Right angled triangle, 80.
Right-hand thread, 117.
Right moment, 126.
Root diameter, 98, 102, 161.
Roots of numbers, 72.

Screw, 136.
" , law of, 137.
Sector, 174.
Shaper, crank, 121.
" , geared, 122.
Sharp "V" thread, 99-102.
Sheave, 130.
Significant figures, 76-79.
Simple geared lathe, 113.
Simple machines, 125.
Spacing of holes, 198.
Specific gravity, 186.
" " , table of, 187.
Speed, 86, 170, 171.
" for finishing, 89.
Spiral milling, 178.
" " , change gears for, 180.
" " , angle for, 181.

Spur gear, 158.
Square root, 73-75.
Square thread, 105.
Supplementary angles, 67.
Systems of payment, 205.

Taper, 90.
" , angle of, 199.
" , Brown and Sharpe, 92.
" calculations, 92-94.
" , Jarno, 91.
" , Morse, 91.
" turning, 33, 90.
Thread, 98.
" , Acme, 108.
" cutting, 34, 112.
" , depth of, 98, 99.
" , diameter of, 98.
" , lead of, 98.
" , left-hand, 99.
" , pitch of, 98.
" , right-hand, 99.
" , sharp "V", 99.
" , square, 105.
" , U. S. Std., 103.
" , Whitworth, 107.
" , worm, 109.
Time cards, 205.
Triangle, 51.
" , draughtsman's, 57.
Trigonometric tables, 193, 195.
Try square, 40.
Tumbling gears, 117.

United States Standard thread, 34, 103.

Vernier, 48.
Vernier caliper, 47.
Vertex, 52.

Volume of solid, 184.

Wedge, 135.
" , law of, 136.
Weight, 125.
" of castings, 188.
" of round bars, 186.
" of sheet iron, 186.
" of square bars, 185.
Weight arm, 126.
Wheel and axle, 129.
" " " , law of, 130.
Whitworth thread, 107.
Work, 143.
Working depth, 160.
Worm, 168.
" thread, 109.
" wheel, 168.

THIS BOOK IS DUE ON THE LAST DATE
STAMPED BELOW

AN INITIAL FINE OF 25 CENTS
WILL BE ASSESSED FOR FAILURE TO RETURN
THIS BOOK ON THE DATE DUE. THE PENALTY
WILL INCREASE TO 50 CENTS ON THE FOURTH
DAY AND TO $1.00 ON THE SEVENTH DAY
OVERDUE.

AUG 10 1934

JUL 19 1938

APR 15 1941 M

A

YB 15840